Due Monday, September 14th, 12:00 midnight

This homework is considering the analysis of plane and space (3D) trusses as discussed in class. In addition to the software for plane trusses used in HW1, a list of MATLAB programs for 3D space trusses is provided together with an example problem.

Start this work by investing considerable time to review the structure and fine details of the provided programs. It is important that all tasks that have been programmed clearly provide you a direct one-to-one link with the theory you learned in lectures.

Problem 1 - Analysis and design of a plane truss (MatLab)

Consider a plane truss as shown in the figure. The horizontal and vertical members have length L, while inclined members have length $\sqrt{2}L$. Assume the Young’s modulus $E = 100$ GPa, cross-sectional area $A = 1.0 \text{ cm}^2$, and $L = 0.3 \text{ m}$.

1. Use the finite element (MatLab) program to determine the tip deflections for the following three load cases (the subscripts refer to finite element nodes).

   - Load Case (A) $F_{x13} = F_{x14} = 10,000N$
   - Load Case (B) $F_{y13} = F_{y14} = 10,000N$
   - Load Case (C) $F_{x13} = 10,000N$ and $F_{x14} = -10,000N$

2. Assuming that the truss behaves like a cantilever beam, one can determine the equivalent cross sectional properties of the beam from the results for Cases A through C above. The three beam properties are: axial rigidity $(EA)_{eq}$ (this is different from the $AE$ of the truss member), flexural rigidity $(EI)_{eq}$ and shear rigidity $(GA)_{eq}$. Let the beam length be equal to $l$. ($l = 6 \times 0.3 = 1.8 \text{ m}$)

The axial deflection of a beam due to an axial force $F$ is given by:

$$u_{tip} = \frac{Fl}{(EA)_{eq}}$$  \hspace{1cm} (1)

The transverse deflection due to a transverse force $F$ at the tip
In Eq. (2) the first term on the RHS represents the deflection due to flexure and the second term due to shear deformation. In the elementary beam theory (Euler-Bernoulli beam theory) we neglect the shear deformation, as it is usually much smaller than the flexural deflection.

The transverse deflection due to an end couple \( C \) is given by

\[
v_{tip} = \frac{Cl^2}{2(EI)_{eq}}
\]  

(3)

Substitute the average tip deflections obtained in Part 1 in Eqs. (1)-(3) to compute the equivalent section properties: \((EA)_{eq}\), \((EI)_{eq}\) and \((GA)_{eq}\).

You may use the average of deflections at Nodes 13 and 14 to determine the equivalent beam deflections.

3. Verify the beam model by adding two more bars to the truss \((l = 8 \times 0.3 = 2.4 \text{ m})\). Compute the tip deflections of the extended truss for the three load cases A-C at the end nodes 17 and 18 using the FE program. Compare the FE results with deflections obtained from the equivalent beam model (Eqs. (1)-(3)).
Problem 2 – Space truss structure (MatLab)

Using the given program for 3D space trusses, determine joint displacements and stresses in the space truss shown in the Figure. Can you do some simple checks to verify that the results are correct?

Problem 3 – Space truss structure (Ansys)

Repeat the solution of Problem 2 using Ansys as described in your recitation. Provide a complete list of the command steps that you used to solve this problem with Ansys and compare your answers with those obtained with MatLab.
Problem 4 - Analysis of trusses with temperature change (MatLab programming required)

All members have the same cross-sectional area and are of the same material, $A = 1 \text{ in}^2$ and $E = 29.5 \times 10^6 \text{ lb/ in}^2$. The horizontal distance is 40 in and the vertical distance is 30 in. The two bars shown experience a temperature rise of $50 \text{ F}$. The coefficient of thermal expansion is $\alpha = 1/150000 \text{ / F}$. Modify the given MatLab programs for plane trusses to account for temperature change and provide the computed nodal displacements, stresses and reaction forces.

**Hint:** As discussed in class (please repeat these calculations in your HW solution), introduction of temperature effects only affects the element load vectors. Programming this should be trivial: In the data of the problem, introduce an index that defines for which element temperature effects should be accounted. Then for these elements, modify the element force vector to account for the initial thermal strain $\varepsilon_0 = \alpha \Delta T$. Once you compute the displacements, please note that the stress calculation will need to be modified for the elements with temperature effects as $\sigma_x = E \left( \frac{du}{dx} - \varepsilon_0 \right)$.

While this is a simple truss, your modified program should be capable of accounting for thermal effects in multiple members of arbitrary two-dimensional trusses.