Multiphase Flow
As in the multiphase flow in vertical pipe, in horizontal pipe there are distinct flow regimes. In horizontal flow there are divided up into 3 main types, Segregated Flow, Intermittent Flow and Distributive Flow. Segregated Flow is divided up into Stratified, Wavy and Annular Flow. Intermittent Flow is divided up into Plug and Slug Flow. Distributive Flow is divided up into Bubble and Mist Flow.

These regimes are predicted by using flow regime maps such as Baker’s shown here.

Horizontal Flow Regimes
Dukler Method

It is based on empirical correlations of friction factor and liquid holdup. Like vertical flow, will assume kinetic energy contribution to be negligible. And there will be no potential energy contribution.

\[
\frac{dp}{dL} = \frac{f \rho_k u_m^2}{2gD} \quad (2-33)
\]

where

\[
\rho_k = \frac{\rho_l \lambda_l^2}{y_l} + \frac{\rho_g \lambda_g^2}{y_g} \quad (2-34)
\]

The friction factor is obtained from the no-slip friction factor, \( f_n \), defined by

\[
f_n = 0.0056 + .5(N_{Re_k})^{-0.32} \quad (2-35)
\]

\[
N_{Re_k} = \frac{1488 \rho_k u_m D}{\mu_m} \quad (2-36)
\]

The 2 phase friction factor is given by the correlation

\[
\frac{f}{f_n} = 1 - \frac{\ln \lambda_l}{1.281 + .478 \ln \lambda_l + .444(\ln \lambda_l)^2 + .094(\ln \lambda_l)^3 + .00843(\ln \lambda_l)^4} \quad (2-37)
\]

In the fig 2-8 the liquid holdup, \( y_l \), is a function of input liquid fraction, \( \lambda_l \) with \( N_{Re_k} \) as a parameter. Since holdup is needed to figure \( N_{Re_k} \), determining the liquid holdup in an iterative process.

Example of Dukler
\begin{align*}
q_o &= 2000 \text{ bpd} \quad q_g = 1 \text{ mmcfpd} \quad \text{Temp} = 175 \, ^\circ \text{F} \\
\mu_o &= 2 \text{ cp} \quad \mu_g = .0131 \quad \text{Pipe} = 2.5" \\
\rho_o &= 49.9 \text{ lb/ft}^3 \quad \rho_g = 2.6 \text{ lb/ft}^3 \quad \text{Pressure} = 800 \text{ psi}
\end{align*}

Find the liquid holdup

1) find the velocities

\begin{align*}
u_l &= \frac{q_o}{A} = \frac{2000 \times 5.614}{.034 \times 86400} = 3.81 \text{ ft/sec} \\
u_g &= \frac{q_g \times z \times T \times p_{sc}}{A \times T_{sc} \times p} \\
&= \frac{1 \times 10^6 \times .935 \times 635 \times 14.7}{.034 \times 520 \times 800 \times 86400} = 7.12 \text{ ft/sec} \\
u_m &= u_l + u_g = 10.94 \text{ ft/sec}
\end{align*}

2) find \( \lambda_l \)

\( \lambda_l = \frac{u_l}{u_m} = 3.81 / 10.94 = .35 \)

2) find liquid holup, \( y_l \)

This is an iterative process

Assume \( y_l = \lambda_l \) and solve for \( \rho_k \).

\begin{align*}
\rho_k &= \frac{\rho_o \lambda_l^2}{y_l} + \frac{\rho_g \lambda_g^2}{y_g} = \frac{49.9 \times .35^2}{.35} + \frac{2.6 \times 65^2}{.65} = 19.19 \text{ lb/ft}^3 \\
N_{Rek} &= \frac{\rho_k u_m D}{\mu_m} = \frac{19.19 \times 10.94 \times .208 \times 1488}{.7085} = 91600.
\end{align*}

This gives a value of .44 for \( y_l \), use this value to calculate a \( \rho_k \)

\begin{align*}
\rho_k &= \frac{\rho_o \lambda_l^2}{y_l} + \frac{\rho_g \lambda_g^2}{y_g} = \frac{49.9 \times .35^2}{.44} + \frac{2.6 \times 65^2}{.56} = 15.86 \text{ lb/ft}^3 \\
N_{Rek} &= \frac{\rho_k u_m D}{\mu_m} = \frac{15.86 \times 10.94 \times .208 \times 1488}{.7085} = 75800
\end{align*}

\( y_l = .46 \)

Calculate \( f_n \)

\begin{align*}
f_n &= 0.0056 + .5(N_{Rek})^{-0.32} = .0056 + .5 \times 75800^{-0.32} = .019
\end{align*}
Find \( f/f_n \)

\[
\frac{f}{f_n} = 1 - \frac{\ln \lambda_i}{1.281 + .478 \ln \lambda_i + .444 (\ln \lambda_i)^2 + .094 (\ln \lambda_i)^3 + .00843 (\ln \lambda_i)^4}
\]

\[
\frac{f}{.019} = 1 - \frac{\ln .35}{1.281 + .478 \ln .35 + .444 (\ln .35)^2 + .094 (\ln .35)^3 + .00843 (\ln .35)^4}
\]

\( f = .036 \)

Find the gradient

\[
\frac{dp}{dL} = \frac{f \rho u_n^2}{2gD} = \frac{.036 \times 15.86 \times 10.92^2}{2 \times 32.17 \times .208} = 5.08 \text{lb/ft}^3 = .035 \text{psi/ft}
\]