Flow in Gas Wells

Assumptions

- Kinetic Energy change is negligible
- Flow is steady state and isothermal
- No work done by the gas flow

R. V. Smith derived the equation for vertical flow

\[
Q = 200,000 \left[ \frac{d^5}{\gamma_g T_a z_a fX} \left( P_2^2 - e^\gamma P_1^2 \right) \frac{S}{e^\gamma - 1} \right]^5
\]  

(2-18)

Q = Flow rate, mscfpd

Ta = Average Temperature, °R

d = ID of pipe, inches

P1 = Sand face pressure, pwf, at depth X, psi

P2 = Well head pressure, THP, psi

S = 0.0375γX/Taza

γg = gas gravity

f = friction factor

This equation also assumes a constant z and temperature.
The gravity of the fluid when a liquid is present is found be the following equation

\[
\gamma = \gamma_g + \frac{4591\gamma_o}{GLR} \frac{\sqrt{GLR}}{1 + 1123/\sqrt{GLR}}
\]  

(2-19)

There is two ways of finding the friction factor required for equation 2-18. The first is using a table using the ID of the tubing and γQ/μ to get the value. Cullender & Binckley showed that the absolute roughness of 0.0006 and 0.00065 will work for all pipe from 1.25 to 8.625”. So they expressed the friction factor in terms of Q, γ, d, & μ,

\[
f = 30.9208 \times 10^{-3} \frac{Q^{0.056} d^{-0.56} \gamma^{-0.065}}{\mu^{-0.065}}
\]  

(2-20)
Moody friction factors

Rewrite equation 2-18 for Moody Friction Factors

\[ p_{nf}^2 - e^s p_{Th}^2 = \frac{6.67 \times 10^{-4} (Q T a z o)^2 (e^s - 1) f}{d^5} \]  
(2-21)

\[ N_{Re} = 20.09 \frac{\gamma Q}{d \mu} \]  
(2-22)

Cullender & Binkley Friction Group

Rewrite equation 2-18 for Fanning Friction Factor

\[ p_{nf}^2 = e^s p_{Th}^2 + R^2 \]  
(2-23)

define R

\[ R^2 = (F_{new} Q)^{9.35} (e^s - 1) \]  
(2-24)

and

\[ F_D = 65.710 / d^{2.614} \]  
(2-25)

\[ F_{Dnew} = F_D \left( \frac{Tz}{540} \right)^{1.0336} \left( \frac{.72 \mu}{.0109 \gamma} \right)^{0.0336} \]  
(2-26)
Poettmann derived an equation to take into account the changing z factor.

\[ X = \frac{X_s}{1 + 0.9521 \times 10^{-6} \times fQ^2G^2X_s^2 / D^5(\Delta P)^2} \]  

\[ X_s = \frac{53.241T_a}{G} \left( \int_{z_2}^{p_2} \frac{z}{P_r} dP_r - \int_{z_2}^{p_2} \frac{z}{P_r} dP_r \right) \]  

Tables are used to solve the integrals in these equations. This method requires for the pressure to be computed at several depths and then plotted pressure vs. depth. The value for the BHP is found from this graph.

To calculate the static bottom hole pressure of a gas well use the following:

\[ P_s - P_{WH} = P_{WH} \left( \exp \left( \frac{0.01877GX}{T_a z_a} \right) - 1 \right) \]  

or

\[ \int_{z_2}^{p_2} \frac{z}{P_r} dP_r = 0.01877 \frac{XG}{T_a} + \int_{z_2}^{p_2} \frac{z}{P_r} dP_r \]  

X is the \( \Delta h \), \( P_1 \) is the wellhead pressure, \( P_2 \) is the bottom hole pressure, psia.
Example using Cullender & Binkley

L = 6830’  2 ½” tubing    gas gravity = .6127    μ = .018 cp
SITHP = 2512 psi    FTHP = 2490 psi    Q = 4.08 mmscfpd

Static pressure

P_a assumed   2730 psi    T_a = 585 R    z_a = .822

\[ s = \frac{0.0375 \gamma L}{T_a z_a} = \frac{0.0375 \cdot 0.6127 \cdot 6830}{585 \cdot 0.822} = .33 \]

\[ P_{th}^2(000's) = 6310 \text{ psi} \]

\[ P_{bh}^2(000's) = e^s P_{th}^2 = e^{.33} 6310 = 8770 \text{ psi}^2 \]

\[ P_{bh} = 2961 \text{ psi} \]

Flowing pressure

P_a assumed   2714 psi    T_a = 598 R    z_a = .835

\[ F_D = \frac{65.710}{d^{2.614}} = \frac{65.710}{2.445^{2.624}} = 6.376 \]

\[ F_{Dnew} = F_D \left( \frac{T_a z_a}{540} \right)^{1.036} \left( \frac{.72 \mu}{.0109 \gamma} \right)^{0.036} = 6.376 \left( \frac{598 \cdot 0.835}{540} \right)^{1.036} \left( \frac{.72 \cdot 0.018}{.0109 \cdot 0.6127} \right)^{0.036} = 6.02 \]

\[ s = \frac{0.0375 \gamma L}{T_a z_a} = \frac{0.0375 \cdot 0.6127 \cdot 6830}{598 \cdot 0.835} = .317 \]

\[ R^2 = (F_{Dnew} Q)^{1.935} \left( e^s - 1 \right) = (6.02 \cdot 4.08)^{1.935} \left( e^{.317} - 1 \right) = 181 \]

\[ P_{wf}^2(000's) = e^s P_{TTH}^2 + R^2 = e^{.317} 6200 + 181 = 8681 \text{ psi}(000's) \]

\[ P_{wf} = 2946 \text{ psi} \]
Wellbore Flow
Summary

Flow regimes

Flow in the wellbore is not single phase in most cases but a multiphase flow of gas and liquid. This breaks down to 5 flow regimes with varying ratios of liquid to gas. Each of these regimes has different effects on the friction generated by the flow up the well.

Methods

Poettman & Carpenter Method

Assumes negligible kinetic energy and external work done by the flow. The energy loss is caused by slippage and frictional effects.

\[
\frac{\Delta p}{\Delta h} = \frac{\rho + \frac{K}{\rho}}{\rho}
\]

where

\[
\bar{K} = \frac{f q^2 M^2}{(7.413 \times 10^{10} D^3)}
\]

and is found from a plot of empirical data using

\[
D_{vp} = (1.4737 \times 10^{-5}) \frac{Mq}{D}
\]

The pressure drop over short lengths of the well is used to find a pressure profile of the well. The \(\Delta p\) should not be more than 10% of the pressure.

Modified Hagedorn & Brown(mH-B)

A two-phase flow correlation using a two phase Reynolds Number. This is for non-bubble flow, Griffith method is used for bubble flow. The input fraction of gas is used to determine if the flow is in the bubble regime, \(\lambda_g < L_B\).
\[
L_b = 1.071 - 0.2218 \left( \frac{u_m^2}{D} \right) \\
\lambda_g = \frac{u_g}{u_m}
\]

Then using 4 dimensionless numbers to find the liquid hold up (\(\psi\)), and the Reynolds number from the correlation graphs. Using the two-phase Reynolds number and the Moody chart to find the friction factor to use in the equation.

\[
144 \frac{dp}{dh} = \tilde{\rho} + \frac{fw^2}{\left(7.413 \times 10^5 D^5\right) \tilde{\rho}}
\]

If the flow is in the bubble flow regime use the Griffith correlation.

\[
144 \frac{dp}{dh} = \tilde{\rho} + \frac{fw_f^2}{\left(7.413 \times 10^5 D^5\right) \rho_f y_f^2}
\]

\[
y_f = 1 - \frac{1}{2} \left[ 1 + \frac{u_m}{u_s} - \sqrt{\left(1 + \frac{u_m}{u_s}\right)^2 - 4 \frac{u_g}{u_s}} \right]
\]

The Reynolds number is calculated by the equations

\[
N_{Re} = \frac{D \bar{u}_l \rho_l}{\mu_l} \\
N_{Re} = \frac{2.2 \bar{E} - 2w_f}{D \mu_l}
\]

**Gilbert Method**

A graphic method using plots for each flow rate, tubing size, oil and gas gravities and temperature. These are plots of pressure vs length, by using equivalent length either the bottom hole pressure can be found from the tubing head pressure, or the tubing head from the bottom hole pressure.