Drilling Hydraulics

\[ P_p = P_s + P_{idp} + P_{ic} + P_{bit} + P_{ac} + P_{adp} \]

Design Criteria
- Must annular velocity high enough to pick up cuttings
  - Need laminar flow
  - To high will also cause hole erosion
- Pressure drop across the bit for maximum efficiency
- Hydraulic Power
- Hydraulic Impact Force

Fluids
- Both Newtonian and Nonnewtonian fluids are used in drilling systems.

Flow regimes
- Laminar in the annulus
- Turbulent in the drill string

Assumptions for flow in pipe and annulus
- Isothermal flow
- Incompressible flow
- Concentric circular annulus
- Nonrotating drill string
- Layers adjacent to walls are stationary
- Shear rate is a function of shear stress at that point
- Steady state flow

\[ N_{Rp} < 2100 \] the flow is Laminar

For Newtonian fluids

\[ N_{kp} = \frac{\rho \bar{v} d_p}{\mu} \]
In Field units

\[ N_{rp} = \frac{928 \rho v d_{ip}}{\mu} \]

\( \rho \) in \#/gal \quad v in ft/sec \quad d_{ip} in inches

For laminar flow in pipe

\[ \Delta P_{fp} = \frac{\mu v_p}{1500d_{ip}^2} \]

\( \mu \) fluid viscosity \( cp \) \quad v_p \ \text{ave velocity in the pipe} \ ft/sec \quad d_{ip} \ \text{Pipe ID inches} \quad \Delta P_{fp} \ \text{friction gradient in the pipe} \ \text{psi/ft}

For laminar flow in the annulus

\[ N_{re} = \frac{757 \rho \bar{v}(d_2 - d_1)}{\mu} \]

\( d_1 \) outside diameter of the pipe \quad d_2 \ diameter of the hole, \ \text{inches} \n
\[ \Delta P_{fa} = \frac{\mu \bar{v}}{1000(d_2 - d_1)^2} \]

\( \Delta P_{fa} \ \text{friction pressure gradient in the annulus} \ \text{psi/ft} \)
Example Data

TD 10,000’       Hole size 8.875”
DP 5” OD 4” ID     Collars 7” OD 4”ID
Mud μ 40 cp       ρ 8.5#/gal
Minimum annular velocity 100 ft/min

Find flow rate Q

\[ Q = \frac{\pi}{4} \left( d_{h}^2 - d_{pod}^2 \right) \]

\[ Q = 100 \frac{\pi}{4} \left( 74.2^2 - .417^2 \right) = 29.3 \text{ ft}^3 / \text{min} \]

\[ Q = 100 \frac{\pi}{4} \left( 8.875^2 - 5^2 \right) .052 \approx 219 \text{ gpm} \]

Since \( d_{iddp} \) is equal to \( d_{idc} \)

\[ \Delta P_{dpc} = \frac{\mu v_p}{1500 d_{id}^2} \]

\[ v_p = \frac{Q}{A_{id}} = \frac{.488 \text{ ft}^3 / \text{sec}}{8.73E - 2 \text{ ft}^2} = 5.6 \text{ ft / sec} \]

\[ \Delta P_{dpc} = \frac{40 \cdot 5.6}{1500 \cdot 4^2} = .009 \text{ psi / ft} \]

\[ P_{sp} = \Delta P_{dpc} TD = .009 \cdot 10000 = 90 \text{ psi} \]

\[ \Delta P_{ac} = \frac{\mu v_p}{1500(d_h^2 - d_{cod}^2)} \]

\[ v_{ac} = \frac{Q}{A_{ac}} = \frac{.488 \text{ ft}^3 / \text{sec}}{.162 \text{ ft}^2} = 3 \text{ ft / sec} \]

\[ \Delta P_{ac} = \frac{40 \cdot 3}{1500(8.875^2 - 7^2)} = .0026 \text{ psi / ft} \]
\[
\Delta P_{adp} = \frac{\mu \nu_p}{1500(d^2_h - d^2_{adp})}
\]

\[
\Delta P_{adp} = \frac{40 \cdot 1.67}{1500(8.875^2 - 5^2)} = 8.27E - 4 \text{ psi/ft}
\]

\[
P = P_{fp} + \Delta P_{ac, LC} + \Delta P_{adp, LDP}
\]

\[
P = 90 + .0026 \cdot 1000 + 8.27E - 4 \cdot 9000 = 100 \text{ psi}
\]

\[
HHP = \frac{QP}{1714} = \frac{219 \cdot 100}{1714} = 12.7
\]

**Homework**

**Data**

- TD 10,000’
- Hole size 9”
- DP 5” OD 4” ID
- Collars 7” OD 3.5” ID
- 1000’ of collars in the drill string
- Fluid is water
- Minimum annular velocity 150 ft/min
For Newtonian Turbulent Flow

\[ N_{re} > 2100 \]

In pipe

\[ \Delta P_f = \frac{f \rho v^2}{25.8d} \]

\[ f \text{ is the fanning friction factor} \]

\[ \frac{1}{\sqrt{f}} = 2.28 - 4 \log \left( \frac{\varepsilon}{d_p} + \frac{21.25}{N_{Re}^{9/4}} \right) \]

\[ f = \frac{0.0791}{N_{Re}^{25}} \] for smooth wall pipe

or

\[ \Delta P_f = \frac{\rho^{75}v^{1.75} \mu^{25}}{1800d^{1.25}} \]

In annulus

\[ \Delta P_f = \frac{f \rho v^2}{21.1(d_h^2 - d_{pod}^2)} \]

or

\[ \Delta P_f = \frac{\rho^{75}v^{1.75} \mu^{25}}{1398(d_h^2 - d_{pod}^2)^{1.25}} \]
Example
Data

<table>
<thead>
<tr>
<th>TD</th>
<th>10,000’</th>
<th>Hole size</th>
<th>8.875”</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP</td>
<td>5”</td>
<td>OD 4” ID</td>
<td>7” OD</td>
</tr>
<tr>
<td>Mud</td>
<td>μ 40 cp</td>
<td>ρ 8.5#/gal</td>
<td></td>
</tr>
<tr>
<td>Pump rate</td>
<td>300 gpm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


\[
v_p = \frac{Q}{A_{in}} = \frac{.668 \text{ ft}^3/\text{sec}}{8.73E-2 \text{ ft}^2} = 7.65 \text{ ft/ sec}
\]

\[
N_{rp} = \frac{928 \rho \nu d_p}{\mu} = \frac{928 \cdot 8.5 \cdot 7.65 \cdot 4}{40} = 6035
\]

\[
f = \frac{0.0791}{N_{re}^{.25}} = \frac{0.0791}{6035^{.25}} = .008
\]

\[
\Delta P_f = \frac{f \rho \nu^2}{25.8 d} = \frac{.008 \cdot 8.5 \cdot 7.65^2}{25.8 \cdot 4} = .043 \text{ psi / ft}
\]

\[
\Delta P_f = \frac{P_d^{.75} \nu^{1.75} \mu^{.25}}{1800d^{1.25}} = \frac{8.5^{.75} \cdot 7.65^{1.75} \cdot 4^{.25}}{1800 \cdot 4^{1.25}} = .043 \text{ psi / ft}
\]

For the annulus
Collars

\[
v_{ac} = \frac{Q}{A_{ac}} = \frac{.668 \text{ ft}^3/\text{sec}}{.162 \text{ ft}^2} = 4.12 \text{ ft / sec}
\]

\[
\Delta P_{ac} = \frac{40 \cdot 4.12}{1500(8.875^2 - 7^2)} = .0037 \text{ psi / ft}
\]
Drill pipe

\[ \nu_{adpc} = \frac{Q}{A_{ac}} = \frac{.488 \text{ ft}^3 / \text{sec}}{.293 \text{ ft}^2} = 1.66 \text{ ft / sec} \]

\[ N_{Re} = \frac{757 \rho \nu(d_2 - d_1)}{\mu} = \frac{757 \cdot 8.5 \cdot 1.66(8.875 - 5)}{40} = 1035 \]

Laminar Flow