Assumptions
1) Change of pressure due to elevation is negligible.
2) Velocity upstream is negligible compared to nozzles.
3) Pressure due to friction is negligible.

\[ \Delta P_B - 8.075E - 4p v_n^2 = 0 \]

\[ \Delta P_B \quad \text{Pressure drop across bit, } \quad v_n \quad \text{nozzle velocity} \]

Solving for nozzle velocity

\[ v_n = \sqrt{\frac{\Delta P_B}{8.074E - 4p}} \]

In the field it has been shown that velocity predicted by this equation is off. So it has been modified,

\[ v_n = C_d \sqrt{\frac{\Delta P_B}{8.074E - 4p}} \]

the recommended valve for \( C_d \) is .95.

If 3 nozzles are present
\[ v = \frac{q_1}{A_1} = \frac{q_2}{A_2} = \frac{q_3}{A_3} \quad \text{the velocity is equal in all the jets.} \]

\[ q = q_1 + q_2 + q_3 = v_n A_1 + v_n A_2 + v_n A_3 \]

That gives us

\[ v_n = q / A_i \quad \text{In field units} \quad v_n = q / 3.117 A_i \]

\( q \) in gpm, \( A_i \) in inches\(^2\), \( v_n \) in ft/sec

solving for the pressure drop

\[ \Delta P_B = \frac{8.311E - 5 \rho q^2}{C_d^2 A_i^2} \]

\( \rho \) is #/gal

Flow Exponent \( \alpha \)

It can be deduced that

\[ P_f = C Q^\alpha \quad C \text{ is a constant} \]

\[ \log P_f = \log C + \alpha \log Q \]

So the log log plot of this equation is a straight line with a slope of \( \alpha \).
\( \alpha \) can found if two \( P_f \) and \( Q \) are known, this can be achieved by measuring the standpipe or surface pressure for 2 pumping rates. \( P_s = P_f + P_B \) so by using the above equation \( P_B \) can be calculated and subtracted from \( P_s \) to find \( P_f \).

\[ P_f = P_s - \frac{8.311E - 5 \rho q^2}{C_d^2 A_i^2} \quad \text{after finding } P_f, \alpha \text{ can be found by} \]
\[
\alpha = \frac{\log\left(\frac{P_f^2}{P_f^1}\right)}{\log\left(\frac{Q^2}{Q^1}\right)}
\]

Maximum Drill Bit Hydraulic Horsepower Criterion assumes that optimum hole cleaning is achieved if the hydraulic horsepower across the bit is maximized with respect to the flow rate \(Q\).

\[H_{HB} = P_b Q\]

Sub in

\[P_b = P_s - CQ^\alpha\]
\[H_{HB} = P_s - CQ^{\alpha+1}\]

Take the first derivative of \(H\) with respect to \(Q\) set the result to 0.

\[\frac{gH_{HB}}{dQ} = P_s - (\alpha + 1)CQ^\alpha = 0\]

\[P_f = CQ^\alpha \quad P_s - (\alpha - 1)P_f = 0 \quad \text{or} \quad P_f = \frac{1}{\alpha + 1} P_s\]

this is the root that makes \(H_{HB}\) a maximum.

Hence the optimum bit hydraulics will be achieved if friction pressure loss in the system is maintained at an optimum value of

\[P_{f_{opt}} = \frac{1}{\alpha + 1} P_{s_{max}}\]

across the nozzles

\[P_{b_{opt}} = P_{s_{max}} - P_{f_{opt}} = \frac{\alpha}{\alpha + 1} P_{s_{max}}\]
Calculate or measure a $P_{fqa}$ @ some $Q_a$ then knowing $P_{fopt}$ a $Q_{opt}$ can be calculated by

$$Q_{opt} = Q_a \, anti \log \left[ \frac{1}{\alpha} \log \left( \frac{P_{fopt}}{P_{fqa}} \right) \right]$$

With $Q_{opt}$ known the $P_{Bopt}$ can be rewritten

$$P_{Bopt} = \frac{8.3E - 5\rho Q_{opt}}{A_{opt}^2 C_d^2}$$

solve for $A_{opt}$

$$A_{opt} = \sqrt{\frac{8.3E - 5\rho Q_{opt}}{C_d^2 P_{Bopt}}}$$

if all nozzles are the same size $A_{opt} = \frac{\pi}{4} n d_{nopt}^2$ $n$ is the number of nozzles

solve for $d_{opt}$

$$d_{opt} = \sqrt{\frac{A_{opt}}{n \pi}}$$
Example:
DP 41/2” 20#/ft, Collars 7” 120.3#/ft 1000’
Mud $\theta_{300} 21$, $\theta_{600} 29$, $\rho 15.5$#/gal
Pump $P_{max} 5440$ psi $H_{HP} 1600$hp 80%
TD 12,000’ $V_{amin} 85$ ft/min
Bit 8 7/8” 14-14-14 Hole size 9 7/8”

Rate data
$Q_1$ 300 GPM @ $P_{f1} 2966$ psi
$Q_2$ 400 GPM @ $P_{f2} 4883$ psi

Find $\alpha$

$$P_B = \frac{8.311E5 \rho Q^2}{C_d A_i^2}$$

$$P_{B1} = \frac{8.311E5 \cdot 15.5 \cdot 300^2}{.95^2 \cdot 45099^2} = 631.6 \text{ psi}$$

$$P_{B2} = \frac{8.311E5 \cdot 15.5 \cdot 400^2}{.95^2 \cdot 45099^2} = 1122.8 \text{ psi}$$

$$P_{f1} = 2966 - 631.6 = 2344.4 \text{ psi}$$

$$P_{f2} = 4883 - 1122.8 = 3760.2 \text{ psi}$$

$$\alpha = \frac{\log\left(\frac{P_{f2}}{P_{f1}}\right)}{\log\left(\frac{Q_2}{Q_1}\right)} = \frac{\log\left(\frac{3760.2}{2344.4}\right)}{\log\left(\frac{400}{300}\right)} = 1.66$$

Find $Q_{max}$ and $Q_{min}$

$$Q_{max} = 1714 \cdot .8 \left(\frac{1600}{5440}\right) = 403 \text{ gpm} \quad \text{Based on pump}$$

$$Q_{min} = 2.448 \left(9.875^2 - 4.5^2\right) \left(\frac{85}{60}\right) = 268 \text{ gpm} \quad \text{Based on velocity}$$

Optimum friction pressure

$$P_{fopt} = \left(\frac{1}{\alpha + 1}\right) P_{max} = \left(\frac{1}{1.66 + 1}\right) 5440 = 2047 \text{ psi}$$
Optimum pressure drop at the bit

\[ P_B = P_{\text{max}} - P_{\text{fopt}} = 5440 - 2047 = 3933 \text{ psi} \]

Optimum flow rate

\[ Q_{\text{opt}} = Q_{\text{anti}} \log \left( \frac{1}{\alpha} \log \left( \frac{P_{\text{fopt}}}{P_{\text{qua}}} \right) \right) = 300\text{anti} \log \left[ \frac{1}{1.66} \log \frac{2047}{2334} \right] = 227 \text{ gpm} \]

This is lower than the max and higher than min flow rates.

Optimum nozzle area

\[ A_{\text{opt}} = \sqrt{\frac{8.311E - 5 \cdot \rho \cdot Q_{\text{opt}}^2}{C_d^2 P_{\text{Bopt}}}} = \sqrt{\frac{8.311E - 5 \cdot 15.5 \cdot 227^2}{.95^2 3393}} = .15 \text{ in}^2 \]

For 3 equal sized jets

\[ d_{\text{opt}} = 2 \sqrt{\frac{15}{3\pi}} = .25 \text{ in} \]

Homework 10/31/08
The rate in gpm to have turbulent flow in the annulus of a 8” hole with 5 ½” 15.5#/ft casing. What will the friction drop be in the casing and annulus if the fluid is water with a TD of 7000’.