Decline Curves
Type Curves
Fetkovich

Fetkovich combined the solutions of the diffusivity and Arps equation to form a generalized method of analyzing decline curves.

Dimensionless values

\[ q_{Dd} = \frac{q_i^2}{q_i} \quad t_{Dt} = D_i t \]

Analytic solution of the diffusivity equation for the boundary-dominated flow behavior in a vertical well.

\[ q_{wD} = \frac{1}{\left( \ln r_e D - \frac{3}{4} \right)} \exp \left[ \frac{-2t_D}{r_e^2 \left( \ln r_e D - \frac{3}{4} \right)} \right] \]

Fetkovich found the ½ fit the field data better. Including definitions for time and radius transforms the equation becomes

\[ q_{wD} = \frac{q}{q_i} \left[ \ln \left( \frac{r_e}{r_{wD}} \right) - \frac{1}{2} \right]^{-1} \]

Using dimensionless flow rate

\[ q_{wD} = q_{Dd} \left[ \ln \left( \frac{r_e}{r_{wD}} \right) - \frac{1}{2} \right]^{-1} \]

This establishes a relationship to the van Everdingen and Hurst flow rate for constant pressure and results in a dimensionless flow rate that can be used to analyze the production data.
\[ q_{Dd} = \frac{141.2qB\mu}{kh(p_i-p_{wf})} \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \]

Take the log of both sides

\[ \ln q_{Dd} = \ln \left[ \frac{141.2qb\mu}{kh(p_i-p_{wf})} \right] + \ln \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \]

This establishes a relationship between Fetkovich dimensionless and field data. The difference is the constant term on the left

\[ \ln \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \]

Can be rearranged to find the formation conductivity

\[ kh = \frac{141.2B\mu}{p_i-p_{wf}} \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \left[ \frac{q}{q_{Dd}} \right]_{MP} \]

A similar equation for gas can be developed, using mscfpd units for the flow rate and in terms of real gas pseudopressure which can be generally used or in pressure squared for reservoirs under 3000 psi.

\[ kh = \frac{1422T}{p_p(p_i-p_{wf})^2} \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \left[ \frac{q}{q_{Dd}} \right]_{MP} \]

\[ kh = \frac{1422T}{p_i^2-p_{wf}^2} \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \left[ \frac{q}{q_{Dd}} \right]_{MP} \]

Combining the material balance equation and the productivity equation

\[ D_i = \frac{q_{i max}}{N_p} \]
This is a function of the theoretical maximum production rate, \( p_{wf} = 0 \) and the oil in place, \( q = 0 \).

\[
q_{i \text{ max}} = \frac{khp_i}{141.2 \mu_i B_{oi} \ln\left(\frac{r_e}{r_w}\right)^{-1/2}}
\]

\[
N_p = \frac{\pi(r_e^2-r_w^2)\phi c_t h p_i}{5.615 B_{oi}}
\]

Combining these definitions with the field

\[
t_{Dd} = \frac{0.00633k t}{\phi \mu c_t r_{wd}^2} \frac{1}{\frac{1}{2}\left(\frac{r_e}{r_{wa}}\right)^2 - 1}\frac{1}{\ln\left(\frac{r_e}{r_{wa}}\right)^{-1/2}}
\]

The relationship between dimensionless time and real time is the constant on the right side of the equation

\[
\frac{1}{\frac{1}{2}\left(\frac{r_e}{r_{wa}}\right)^2 - 1}\frac{1}{\ln\left(\frac{r_e}{r_{wa}}\right)^{-1/2}}
\]

To estimate the volume of an oilwell drainage area by rearranging \( N_p \), substituting the definition of reservoir pore volume, units are reservoir barrels

\[
V_p = \frac{\pi(r_e^2-r_w^2)\phi h}{5.615} = \left[\frac{B_{oi} \bar{\mu}_{oi}}{\mu_{oi} \bar{c}_t (p_i - p_{wf})}\right] \frac{t}{t_{Dd}} \frac{q}{q_{Dd}}
\]

For gas wells

\[
V_p = \frac{\pi(r_e^2-r_w^2)\phi h}{1} = \left[\frac{56.557 T}{\mu_{gi} \bar{c}_t (p_p (p_i) - p_p (p_{wf}))}\right] \frac{t}{t_{Dd}} \frac{q}{q_{Dd}}
\]
\[ V_p = \frac{\pi (r_e^2 - r_w^2) \phi h}{1} = \left[ \frac{56.557T \bar{\mu} \bar{Z}}{\mu_g t_{ct}(p_i^2 - p_{wf}^2)} \right] \left[ \frac{t}{t_{DD}} \right]_{MP} \left[ \frac{q}{q_{DD}} \right]_{MP} \]

In most cases the viscosity in the numerator and the denominator cancel each other out. Also the difference in the initial and current viscosity are negligible for type curve purposes. The FVF is evaluated at the midway pressure, however it is normal to assumed to be equal to the initial pressure FVF. This works well for black oil and high pressure gas reservoirs. But this could introduce inaccuracies in low pressure gas reservoirs. In this case be careful when selecting the proper fluid properties for gas reservoirs with large pressure drawdowns, and high GOR oil wells.

The type-curve stem selected form the match with the field data defines \( r_e/r_w \) value. Rearrange the \( t_{DD} \) equation to a form that can be used to determine the apparent wellbore radius

\[
\begin{align*}
    r_wa &= \left[ \frac{0.0127k}{\phi \mu c_t(r_{eD})^2_{MP} - 1} \left[ \ln\left(\frac{r_{eD}}{r_w}\right)_{MP} - \frac{1}{2} \right] \left[ \frac{t}{t_{DD}} \right]_{MP} \right]^{.5} 
\end{align*}
\]

This \( r_wa \) combined with the S equation to calculate the relation between apparent and mechanical wellbore radius in terms of skin factor.