CHAPTER 9: TREATING EXPERIMENTAL DATA: ERRORS, MISTAKES AND SIGNIFICANCE (Written by Dr. Robert Bretz)

In taking physical measurements, the true value is never known with certainty; the value obtained in a measurement is an approximation of the true value. The difference between the true value and the value taken from the measurement is known as an *error*. Errors associated with a measurement may be categorized as *systematic* or *random* or both. In either case, errors must be assessed and considered when using physical data for engineering purposes as this data is often used for determining size of equipment or for making economic decisions.

Two terms related to measurement error need mentioning: *accuracy* and *precision*. If the error is small compared to the magnitude of the measured quantity, the measurement is said to be accurate. Because the true value of a measured quantity is often unknown, the mean of repetitive measurements is used to represent the magnitude of the measured quantity. The difference between the mean and the observed value is called the *residual*. If the residuals for the repetitive measurements have small value compared to the magnitude of the measured quantity, the measurement is said to be precise. Note that a precise measurement may not be an accurate measurement, but an accurate measurement must be a precise measurement. It is necessary that the precision of measure be reflected when reporting values of physical measurements.

**Systematic errors** have many causes and are especially insidious as their value may not be determined by repeated measurements. A simple example of a systematic error might be making length measurements with a metal ruler. At a particular temperature, the distances indicated are as exact as can be achieved with that ruler. At any other temperature, a correction factor to correct for thermal elongation must be used to express the “correct” value. If the correction is not used, a systematic error in the measurement will occur. Obviously, the systematic error may be insignificant and if so, may be ignored. The ability to judge when corrections are necessary is an important talent to develop in an engineer. Proper calibration and careful consideration of all possible corrections are the only means to prevent systematic errors. Some systematic errors can
be detected because the values drift upon repetition of the measurement. One important method of detecting systematic errors is by comparing reported values from a standard to the presently measurement values. **It is for this reason that it is important to compare the values measured in your experiments in this class to previously reported values for similar measurements if at all possible!!!!!!** Some call systematic errors mistakes.

**Random errors** are indicated by random fluctuations in the value observed in successive measurements. For a given measuring system, these errors are beyond the control of the person making the measurements. Random errors will fluctuate about a mean value, which, if there is no systematic error, will approximate the true value and may be evaluated using statistical methods. To make a long story short, the best approximation of the true value masked by random error is the arithmetic mean.

As noted above, the precision of a measurement may be determined by repetitive measurements to find how values scatter about the mean. An adequate number of repetitions to instill confidence in the statistical result may be impractical or not even possible for some situations. In those cases, the precision of the measurement may be estimated. For instance, in making a length measurement with a ruler the precision is certainly better than the width of the smallest gradation; how much better will depend on the size of the ruler and the eyes of the measurer and his/her ability to judge interim distances. The maximum value of the estimated error is the quantity of most interest in our work.

Most of the physical properties measured in these laboratory exercises are derived from more fundamental measurements such length, time, etc. Determining these properties requires some mathematical manipulation and the way errors combine in these manipulations is important for estimate the maximum error one may expect in the value. The analysis of the affect of individual measurement error on the final physical data value requires that the magnitude of the error approximate a differential amount. Given that assumption we see that if the manipulation is addition or subtraction, by differential calculus,
\[ u = x + y \]
\[ du = dx + dy \] \hfill (9-1)

In terms of finite increments,

\[ \Delta u = \Delta x + \Delta y \] \hfill (9-2)

For multiplication or division,

\[ u = xy \]
\[ du = xdy + ydx \] \hfill (9-3)
\[ \frac{du}{u} = \frac{dy}{y} + \frac{dx}{x} \]

In finite increment terms,

\[ \frac{\Delta u}{u} = \frac{\Delta y}{y} + \frac{\Delta x}{x} \] \hfill (9-4)

Or expressed in percentages,

\[ \frac{\Delta u}{u}100 = \frac{\Delta y}{y}100 + \frac{\Delta x}{x}100 \] \hfill (9-5)

If the property involves logarithms,

\[ u = \ln x \]
\[ du = \frac{dx}{x} \] \hfill (9-6)
\[ \Delta u = \frac{\Delta x}{x} \]
It is important to keep in mind that the above relationships for error only apply if the value of the error is small.

When many repetitions are made, the precision may be analyzed statistically. Common expressions are:

**Average error**, 

\[
a = \pm \frac{\sum (d)}{n}
\]

where \(a\) is the average error, \(d\) is the value of a single deviation, and \(n\) is the number of measurements.

**Standard deviation**, 

\[
\delta = \pm \sqrt{\frac{\sum (d)^2}{n-1}}
\]

Plus and minus one standard deviation will encompass ---- percent of all random errors.

The number of significant figures should express the accuracy of the measurement. When numbers are manipulated using an electronic calculator or computer, it is common for more significant figures than warranted by the accuracy of the measurement to be presented. In a report or other defining document, it is important to round off the result to accurately reflect the accuracy of the measurement. If a measurement yields a value, properly presented with respect significance, of 33, then the actual value will be assumed to lie within values of 32.7 and 33.3 unless an indication of accuracy accompanies the value. A value of 30 is somewhat ambiguous as the “0” may or may not be significant. One may eliminate the ambiguity by writing “30.” for those cases when the “0” is significant. However, leaving off the decimal does not universally mean that the “0” is not significant. Certainly the value 300 adds to the possible confusion as
there is now question as to whether or not either “0” is significant. For the value 30.0, the “0” following the decimal should be significant, otherwise one should write 30 to eliminate confusion.

Bibliography