4.2 Resistivity

The presence of hydrocarbons is identified by the electrical resistance of the formation. These electrical properties of rocks depend on the pore geometry and fluid distribution. That is, the size, type and interconnection of the pore space and the type and distribution of the fluids in the pore space. Rock matrix act as a perfect insulator conducting no electrical current; therefore all conduction is through the fluids in the pore space. This conductivity is known as ionic or electrolytic conduction produced by the movement of ions in the formation water. The ions are typically Na+ and Cl- residing in the water phase. Subsequently, the greater the salinity of the water the higher the conductivity since more ions are present to transmit the current. Oil and gas behave as insulators, therefore a hydrocarbon-bearing formation will exhibit lower conductivity than a water-bearing formation of the same porosity.

The electrical properties of rocks and the pore geometry are combined into Archie’s Law which describes the volume fraction of fluids within the pore space. Understanding this equation is the cornerstone to interpreting well logs.

Fundamentals

To begin our understanding of this process, we must first establish fundamental concepts and clarify terms. To illustrate this concept consider an open-top cubic tank one meter in all dimensions. It is electrically nonconducting except for two opposing metal electrodes (Fig. 4.8). First the tank is filled with water containing sodium chloride to simulate average formation water. A voltage (v) is applied and the resulting current (i) is measured.

Figure 4.8 Definition of water resistivity
Using Ohm's Law the resistance \( r_w \) of the water can be calculated,

\[
r_w = \frac{v}{i}
\]

The resistivity is defined as the resistance scaled by the aspect ratio of area to length,

\[
R_w = r_w \frac{A}{L}
\]

where \( R_w \) is in units of ohm-meter. This resistivity is inversely proportional to water salinity and temperature; i.e., as either property is increased the resultant resistivity will decrease.

Next, sand is added to the water-filled tank, displacing some of the water. The result is a porous media 100% saturated with water. Since the rock grains act as an insulator, the current will be reduced and therefore the resistance will increase. This resistance with respect to the porous rock and associated water content is known as \( r_o \), and the resistivity as \( R_o \).

\[\text{Figure 4.9 Definition of a 100% water saturated sand}\]

The increased resistance is due to the tortuous path the electrical current must take to circumvent the sand grains. This tortuous path length is defined as \( L_a \) and the corresponding cross-sectional area as \( A_a \). The resistivity, \( R_o \), is proportional to \( R_w \) since only the water conducts electricity, thus

\[
F = \frac{R_o}{R_w}
\]
where the proportionality constant $F$ is known as the formation resistivity factor. A plot of formation factor vs. water conductivity is a horizontal line as shown in Figure 4.10; thus $F$ is independent of the formation water resistivity.

![Figure 4.10 Relationship between F and water conductivity](image)

The ratio of resistivities can also be expressed in terms of the aspect ratios:

$$\frac{R_o}{R_w} = \left(\frac{A}{A_a}\right) \left(\frac{L_a}{L}\right)$$

(4.7)

where $r_o \approx r_w$ for the parallel circuit developed in the water-sand mixture. If we define tortuosity ($\tau$) as:

$$\tau = \left(\frac{L_a}{L}\right)^2$$

(4.8)

and porosity ($\phi$) as:

$$\phi = \frac{A_a}{A}$$

(4.9)

then the formation factor can be expressed in terms of variables which represent the pore geometry.

$$F = \frac{\sqrt{\tau}}{\phi}$$

(4.10)
Example 4.3
Consider a synthetic rock sample made of an insulator material and shaped as a cube of length \(L\) as shown below. There is a square tube of dimension \(L/2\) through the cube. Assume the inner square tube is filled with brine of resistivity, \(R_w\) and that the current will flow perpendicular to the faces.
Calculate \(F\), the relationship between \(F\) and porosity, and porosity.

Solution
1. The expression for formation factor is,

\[
F = \frac{\frac{R_o}{R_w}}{\frac{r_o L}{r_w A}} = \frac{A}{A_a} = \frac{\frac{L^2}{L_a}}{\left(\frac{L/2}{L}\right)^2} = 4
\]

In this simple geometry the tortuous path length is equal to the cube length \((L_a = L)\), therefore the only difference is the cross-sectional area.

2. Since the lengths are equal, then \(\tau = 1\) and the formation-factor – porosity relationship is:

\[
F = \frac{1}{\phi}
\]

The result is that porosity is 25%.

3. To verify this result, calculate porosity in terms of the pore volume/bulk volume ratio.
This is in agreement with the previous results.

The previous derivation applies to theoretical models of simple geometry. This approach develops an awareness of the fundamentals in petrophysics; however is not practical in application. The theoretical models do not capture the variations in the porous media found in the real world. An alternative method is to directly measure the formation factor and porosity of rock samples in a laboratory. Figure 4.11 is an example from a sandstone formation.

\[
\phi = \frac{V_p}{V_b} = \frac{(L/2)^2 L}{L^3} = \frac{1}{4}
\]

Figure 4.11 Formation factor – porosity relationship from core samples, [Helander,1983]

Lab-measured values provide excellent results for a particular rock type. However, they are limited to specific formations where core has been retrieved and analyzed in the lab. Frequently, core is not available for the formation of interest; therefore we rely on empirical
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correlations developed in the past. Archie developed a general relationship between porosity and formation factor;

\[ F = a \phi^{-m} \]  

(4.11)

where \( a \) is defined as the cementation factor and \( m \) as the cementation exponent. The term cementation was proposed by Archie because he noticed a dependency of the exponent \( m \) on the degree of cementation (Fig. 4.12).

![Figure 4.12 F-\( \phi \) relationship for various degrees of cementation, [Archie,1942]](image)

Note that for an increase in cementation from unconsolidated to highly cemented rocks the exponent increases from 1.3 to 2.2. For a formation factor of 20 this variation translates into a change in porosity of 10 to 26%, respectively. Subsequent work has shown that the empirical constants are dependent on numerous variables such as cementation, tortuosity, and wettability.

Many empirical correlations have been developed in the past; but only several have proven to be effective over a wide range of formations. Accepted relationships developed in the past are functions of specific rock types; for example, for crystalline type rocks, e.g., limestones and dolomites from Archie;
and for granular type rocks; e.g., sandstones, from Winsauer et al.;

\[ F = 0.62 \phi^{-2.15} \]  \hspace{1cm} (4.13)

or for granular type rocks from Tixier;

\[ F = \frac{0.81}{\phi^2} \]  \hspace{1cm} (4.14)

Both the Tixier and Winsauer equations were derived from the same data set, only different regression analysis. Figure 4.13 provides a comparison of the above relationships. Notice within the normal porosity range of 5 to 20% the three correlations are in good agreement. Also shown on the figure is the effect of secondary porosity; i.e., vugs and fractures. Theoretically, \( a \) should equal one; therefore, when \( \phi = 1 \) (all water no matrix) then \( R_w = R_o \), and when \( \phi = 0 \) (all rock) then \( F \to \infty \).
Discussions up to this point have focused on water resistivity and 100% water saturated resistivity of a porous media. Consider an appreciable fraction of the pore space is filled with hydrocarbons as shown in Figure 4.14. The resistivity of this hydrocarbon-bearing formation is $R_t$. The addition of oil and gas in the pore spaces act as insulators, therefore the resistivity will be much greater than in a water zone.

![Figure 4.14](image)

**Figure 4.14** Resistance in a hydrocarbon-bearing formation

The ratio of the resistivity in a hydrocarbon-bearing formation to the resistivity of a formation 100% saturated with water is known as the Resistivity Index, $I$.

$$I = \frac{R_t}{R_o}$$  \hspace{1cm} (4.15)

Archie correlated experimental data and developed a relationship between Resistivity Index and water saturation ($S_w$),

$$I = \frac{R_t}{R_o} = \frac{1}{S_w^n}$$  \hspace{1cm} (4.16)

where $n$ is known as the saturation exponent. When $S_w = 1$ (all water in pores) then $R_t = R_o$, and when $S_w = 0$ (all oil in pores) then $R_t \to \infty$; thus validating Eq. (4.16). Figure 4.15 is the experimental data evaluated by Archie. The saturation exponent is equal to 2 for Archie’s data. This is the commonly accepted value in practice, however observed variations in $n$ have been attributed to wettability, rock texture, measurement techniques and the presence of clays.
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Figure 4.15 1– $S_w$ relationship from experimental data

Rearranging Equation (4.16) to solve for water saturation and applying $n=2$, results in the following saturation equation.

$$S_w = \sqrt{\frac{R_o}{R_t}}$$  \hspace{1cm} (4.17)

Equation (4.17) is applicable only when a hydrocarbon zone overlays an obvious water-bearing zone of the same porosity and water salinity.

Archie extended Eq. (4.17) to be used for any permeable, hydrocarbon zone by the substitution of Eq. (4.6) for $R_o$, resulting in the following expression,

$$S_w = \sqrt{\frac{F R_w}{R_t}}$$  \hspace{1cm} (4.18)
which is known as Archie's Relation or Archie's Law. *The entire well logging industry is built around this equation!*

**Example 4.4**

Porosity and resistivity data for four zones is given in the table below. Determine the water saturation for the intervals A, B, C and D. Zones A and B are adjacent, Zones C and D are separated by shale layers.

<table>
<thead>
<tr>
<th>ZONE</th>
<th>POROSITY, %</th>
<th>R_T, OHM-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30.0</td>
<td>4.0</td>
</tr>
<tr>
<td>B</td>
<td>30.0</td>
<td>0.4</td>
</tr>
<tr>
<td>C</td>
<td>7.0</td>
<td>8.0</td>
</tr>
<tr>
<td>D</td>
<td>35.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Solution**

Intervals A and B constitute a formation of uniform porosity with an apparent water-oil contact at approximately 7,300 ft. Assuming Zone B is completely water saturated than $R_o$ from Zone B is 0.4 ohm-m, while $R_t$ from Zone A is 4 ohm-m. From Eq. (4.17) the water saturation for zone A is;

$$S_w = \sqrt{\frac{0.4}{4}} = 0.32$$

At Zone C, there is no obvious measurement of $R_o$. The alternative is to apply Archie's relation (Eq. 4.18) to a nearby water-bearing zone and solve for formation water resistivity. Applying this $R_w$ to Zone C is permissible because water salinity and temperature change slowly with depth. Zone D is a nearby water-bearing interval, which results in $R_w = 0.045$ ohm-m. Substituting this value of $R_w$ for zone C yields;

$$S_w = \sqrt{\frac{0.81(0.045)}{8(0.07)^2}} = 0.96$$

4.21
Consequently, this zone is simply a tight water-bearing sandstone. As a final step, it is possible to verify $R_w$ with Archie's relation in Zone A.

$$S_w = \frac{.81(0.045)}{4(0.30)^2} = 0.32$$