FRACTURE

METE 327
Fall 2008
OUTLINE

• What is Fracture?
• The Griffith Equation
• The Orowan Modification
• Statistics—Weibull Distribution
• Examples
  – 35 MgO samples
  – Wesgo Al95 Alumina
  – Plaster
• Transition to Fracture Mechanics
What is Fracture?

- Most Fracture failures are “unexpected” failures

- What is an “unexpected” failure?

- The expected load carrying capacity of a structural member is its yield strength times its cross sectional area

- The presence of a flaw can change this

- The following approach was proposed by Griffith in 1920
The Griffith Equation

A plate of unit thickness has a through crack of length 2c. The crack has a surface energy of 4γ, where γ is the energy required to create a unit area of new surface.

The total energy content of the plate, under the applied stress is σ²/E. Due to the crack, this energy is reduced by the volume of material in the cylinder of radius c.

This energy is \( \pi c^2 \sigma^2 / E \). Now, if the crack extends a small amount, δ, then the strain energy is further reduced by an amount \( (\pi c^2 \sigma^2 / E) \delta \). The additional surface is 4 δ so the additional surface energy is 4 δγ.
The Griffith Equation (cont)

We set these two quantities equal to one another

So, $4\gamma = 2\pi c \delta \sigma^2 / E$. The deltas cancel, and so the stress at which the crack will propagate is:

$$\sigma = \sqrt{2E\gamma / \pi c}$$

And that is the Griffith Equation.
Orowan Modification

For ductile materials, Orowan modified the Griffith equation in 1952 by substituting two terms for $\gamma$: $\gamma_s$ is the surface energy, and $\gamma_p$ is the plastic energy. $\gamma_s$ is about 1-2 J/m$^2$, and $\gamma_p$ is 100-1000 J/m$^2$, so we can write: 

$$\sigma = \sqrt{\gamma_p E/c}$$

Before we use this equation as a lead-in to Fracture Mechanics, some additional aspects of brittle fracture:

Statistical nature—Weibull Distribution
The Weibull Distribution

- Probability of failure, $S = n/(N+1)$ where $N$ is the total number of specimens and $n$ is the order number in increasing strength

- Then $S_i = (1-e^{-B_i})$ where $B_i$ is called the risk of rupture, and $B_i = ((\sigma_i - \sigma_u)/\sigma_o)^m$

- Where $\sigma_i$ is the $i$th specimen strength, $\sigma_u$ is the so-called “zero strength”, $\sigma_o$ is a normalizing factor and $m$ is the flaw density exponent.

- Sometimes $B$ is multiplied by a dimensionless volume to account for a possible size effect.
Weibull Distribution (cont.)

- Take the double logarithm of both sides and plot \( \log \log S \) vs. \( \log (\sigma_i - \sigma_u) \) for different values of \( \sigma_u \).
- The best value of \( \sigma_u \) will give a straight line of slope \( m \).
- A computer program was written to find the best fit using different choices of \( \sigma_u \). (minimize the sum of the deviations squared from a line).
35 MgO Strength Values in increasing order

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23700</td>
</tr>
<tr>
<td>2</td>
<td>23950</td>
</tr>
<tr>
<td>3</td>
<td>25050</td>
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<tr>
<td>4</td>
<td>25800</td>
</tr>
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<td>5</td>
<td>25950</td>
</tr>
<tr>
<td>6</td>
<td>26900</td>
</tr>
<tr>
<td>7</td>
<td>26950</td>
</tr>
<tr>
<td>8</td>
<td>27050</td>
</tr>
<tr>
<td>9</td>
<td>27800</td>
</tr>
<tr>
<td>10</td>
<td>28050</td>
</tr>
<tr>
<td>11</td>
<td>28150</td>
</tr>
<tr>
<td>12</td>
<td>28200</td>
</tr>
<tr>
<td>13</td>
<td>28300</td>
</tr>
<tr>
<td>14</td>
<td>28400</td>
</tr>
<tr>
<td>15</td>
<td>28550</td>
</tr>
<tr>
<td>16</td>
<td>28800</td>
</tr>
<tr>
<td>17</td>
<td>29000</td>
</tr>
<tr>
<td>18</td>
<td>29300</td>
</tr>
</tbody>
</table>
FIGURE 2. Distribution Curve of Strengths of 35 MgO Specimens

- EXPERIMENTAL
- CALCULATED USING THE FOLLOWING
PARAMETERS:

\[ \sigma_u = 12,000 \text{ PSI} \]
\[ m = 7.07 \]
\[ V = 0.00230 \text{ in}^3 \]
\[ \sigma_0 = 4858 \text{ PSI} \]
FIGURE 1. Weibull Plot for 35 MgO Specimens Indicating Computer-Selected Best Straight Line

Yielding: $\sigma_u = 12,000$ PSI
$m + 1 = \text{Slope} = 8.07$
### TABLE 2

<table>
<thead>
<tr>
<th>SPECIMEN GROUP</th>
<th>AVERAGE STRENGTH (PSI)</th>
<th>ZERO STRENGTH (\sigma_u) (PSI)</th>
<th>FLAW DENSITY EXPONENT, (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>As Received</td>
<td>28,485</td>
<td>13,300 (10,000) (^k)</td>
<td>3.21 (3.23)</td>
</tr>
<tr>
<td>As Received Ground</td>
<td>24,972</td>
<td>13,500 (10,000)</td>
<td>3.31 (6.1)</td>
</tr>
<tr>
<td>Annealed</td>
<td>30,618</td>
<td>2,400 (0)</td>
<td>8.36 (10.0)</td>
</tr>
<tr>
<td>Annealed Ground</td>
<td>28,322</td>
<td>0 (0)</td>
<td>12.24 (12.5)</td>
</tr>
</tbody>
</table>

\(^k\)Numbers in parentheses are graphically determined parameters.
FIGURE 3. Distribution Curves of Strengths for Four Wesgo A1995 Populations
<table>
<thead>
<tr>
<th>SPECIMEN GROUP</th>
<th>AVERAGE STRENGTH (PSI)</th>
<th>ZERO STRENGTH $\sigma_u$ (PSI)</th>
<th>FLAW DENSITY EXPONENT, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, C, and D</td>
<td>27,297</td>
<td>0</td>
<td>14.36</td>
</tr>
<tr>
<td>1000 psi/sec</td>
<td>27,297</td>
<td>0</td>
<td>14.36</td>
</tr>
<tr>
<td>3000 psi/sec</td>
<td>30,017</td>
<td>23,500</td>
<td>4.741</td>
</tr>
<tr>
<td>4200 psi/sec</td>
<td>27,987</td>
<td>12,150</td>
<td>7.611</td>
</tr>
<tr>
<td>4500 psi/sec</td>
<td>28,634</td>
<td>0</td>
<td>16.571</td>
</tr>
<tr>
<td><strong>ALL LOAD RATES</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>28,357</td>
<td>0</td>
<td>15.92</td>
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<tr>
<td>B</td>
<td>28,773</td>
<td>100</td>
<td>19.84</td>
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<tr>
<td>C</td>
<td>29,592</td>
<td>15,000</td>
<td>8.78</td>
</tr>
<tr>
<td>D</td>
<td>27,813</td>
<td>17,000</td>
<td>4.10</td>
</tr>
<tr>
<td><strong>ALL DATA</strong></td>
<td>28,655</td>
<td>4,000</td>
<td>13.54</td>
</tr>
<tr>
<td><strong>ARF BEND DATA</strong></td>
<td>28,486</td>
<td>13,300</td>
<td>3.21</td>
</tr>
</tbody>
</table>
### TABLE 5

<table>
<thead>
<tr>
<th>Number of Specimens</th>
<th>Average Strength (PSI)</th>
<th>Zero Strength ($\sigma_u$) (PSI)</th>
<th>Flaw Density Exponent, $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>182</td>
<td>843</td>
<td>270</td>
<td>5.79</td>
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<tr>
<td>50 1.</td>
<td>837</td>
<td>400</td>
<td>3.32</td>
</tr>
<tr>
<td>50 1.</td>
<td>819</td>
<td>0</td>
<td>9.90</td>
</tr>
<tr>
<td>82 1.</td>
<td>861</td>
<td>0</td>
<td>9.40</td>
</tr>
<tr>
<td>100 2.</td>
<td>840</td>
<td>142</td>
<td>7.43</td>
</tr>
<tr>
<td>180</td>
<td>842</td>
<td>200</td>
<td>6.78</td>
</tr>
<tr>
<td>179</td>
<td>840</td>
<td>230</td>
<td>6.37</td>
</tr>
<tr>
<td>178</td>
<td>843</td>
<td>190</td>
<td>6.89</td>
</tr>
<tr>
<td>177</td>
<td>841</td>
<td>230</td>
<td>6.34</td>
</tr>
<tr>
<td>150</td>
<td>843</td>
<td>150</td>
<td>7.52</td>
</tr>
<tr>
<td>130</td>
<td>841</td>
<td>100</td>
<td>8.36</td>
</tr>
</tbody>
</table>

1. Three populations selected by hand without replacement.

2. The results are the average of 20 such selections of 100 specimens each. Three values of $\sigma_u = 0$ were found.
Fracture Mechanics

Earlier we derived the Orowan modification to the Griffith equation, to take into account plastic deformation as an energy absorbing mechanism in crack propagation:

$$\sigma = \sqrt{\gamma_p E / c}$$

George Irwin, in the early 1960's proposed a quantity $G$, the strain energy release rate, or the crack extension force, giving an equation for the fracture stress in terms of $G$:

$$\sigma_f = \sqrt{EG_c / \pi c}$$
but, \(\gamma_p\) and \(G_c\) are difficult quantities to measure, so Irwin proposed the stress intensity factor, \(K = \sigma \sqrt{\pi a}\) where \(\sigma\) is the stress and \(a\) is the half crack length. It is similar to a stress concentration factor, and can be easily measured. \(K\) has somewhat unusual units of Mpa-m\(^{1/2}\) or KSI-in\(^{1/2}\). Now, 
\[G = \pi c \sigma^2 / E\] and \(K^2 = GE\) so, substituting we get:
\[K = \sigma \sqrt{\pi c}\]
or, more generally, where \(\alpha\) includes a number of geometrical factors:
\[K = \alpha \sigma \sqrt{\pi c}\]
The Critical Stress Intensity Factor

- Usually expressed as $K_{lc}$, and it is a true material property
- There is a minimum thickness in order to be certain that plane strain conditions exist
  - Thickness, $B = 2.5\left(\frac{K_{lc}}{\sigma_0}\right)^2$ is the criterion where $\sigma_0$ is the 0.2% offset yield strength
- There can be no assurance that a test will be valid until it is performed
- An estimate of thickness can be made using an expected value of $K_{lc}$
Crack Opening Modes
Figure 11-7 Effect of specimen thickness on stress and mode of fracture. *(From C. C. Osgood, Machine Design, August 5, 1971, p. 91.)*
Types of Test Specimens

Compact tension specimen

Bend specimen
Types of Test Specimens (cont.)

\[ K_I = \frac{P}{D^{3/2}} \left[ 1.72 \left( \frac{D}{d} \right) - 1.27 \right] \]

Notched round specimen
Different types of load-displacement curves

**Figure 11.9** Load-displacement curves. (Note that slope $OP_3$ is exaggerated for clarity.)
Explanation of Curves

- Type I—typical of most ductile metals. Line OP is at 5% lower slope than tangent OA. \( P_s = P_Q \)
  - If \( x_1 \) is more than \( \frac{1}{4} \) of \( x_s \) then material is too ductile

- Type II—shows a pop-in, but must meet the maximum ductility criterion—\( P_Q \) is max load

- Type III—shows a complete pop-in instability and is characteristic of a brittle “elastic” material

- \( P_Q \) is used to calculate \( K_Q \) and if the thickness is greater than calculated for plane strain, then \( K_Q \) equals \( K_{IC} \).
Example: Thin-Wall Pressure Vessel

\[ \sigma = \frac{PD}{2t} \]

\[ k_I^2 = \frac{1.21a \pi \sigma^2}{Q} \]

Figure 11-5 Flaw geometry and design of cylindrical pressure vessel.
Thin-Wall Pressure Vessel

- Material: Ti 6Al 4V, Hoop Stress = 360 MPa
- $K_{lc} = 57$ Mpa m$^{1/2}$
- $\sigma_o = 900$ Mpa
- Crack oriented as shown above
- For this type of loading and geometry:
  - $K_i^2 = (1$.21a$\pi \sigma^2)/Q$
  - $a =$ Surface crack depth
Thin-Wall Pressure Vessel

- For a wall thickness of 12 mm, and $\sigma/\sigma_0 = 0.4$
- If $2c=2a$ then $Q = 2.35$
- Then the critical crack size, $a_c = 15.5$ mm
- The critical crack depth is greater than the wall thickness and the vessel will “leak before burst”
- However if $a/2c = 0.05$, $Q = 1$ and $a_c = 6.6$ mm
- This is less than the wall thickness
- So, vessel will burst!
Figure 11-18 Range of toughness and yield strength values for a variety of alloys at room temperature
(From Rapid Inexpensive Tests for Determining Fracture Toughness, NMAB Report 328, 1976.)
Figure 4  The Variable Sweep Wing of the F-111 Employed D6ac Steel Components for Load Transfer
Steel, compression side

Crack growth by fatigue

1"

Pre-existing crack

Steel, tension side
Figure 6 Application of Boron Doubler Reduced Stress Levels in Wing Pivot Fittings

Limit Load
- Doubler/Strain
- $L/H = 5550$ in./in.
- Required Bond Shear Stress = 1100 psi calculated
Figure 9.14 (a) Fatigue striations typical of an unembrittled alloy steel (x 4000).
(b) Intergranular facets (f) and striations (S) in temper-embrittled alloy steel (x 350)
courtesy of R. O. Ritchie
Figure 9.4 Schematic representation of fatigue crack advance by plastic blunting process
Figure 9.9 Basic data: crack length v. number of cycles
Figure 9.10 Schematic crack growth-rate dependence on $K$
Influence of corrosive environment

Figure 9.10 Schematic crack growth-rate dependence on $K$
Figure 9.13 (a) Effect of mean stress on fatigue crack length in low-alloy steel ($\Delta K_i = 15$ MN m$^{3/2}$). (b) Effect of mean stress on crack growth-rate and apparent slope of growth-rate curve. Apparent slopes obtained by least mean squares regression analysis (after Ritchie and Knott 14).
Cyclic Loading

Spectrum Loading (Aircraft)
Periodic Overloads

Lifetime

N

N

Time
Periodic Overloads

Notch Root

Extent of flat fracture
Periodic Overloads

- The overload creates a larger plastic zone
- If applied infrequently, the residual stress does not build up to slow the crack propagation
- If applied to often, it causes a greater rate of crack propagation
- At an intermediate rate, the plastic zone, and hence the residual compressive stress, is present for more of the growth cycles
Plastic Zone Size

\[ r_p = \frac{1}{2\pi} \left( \frac{\sigma^2 a}{\sigma_0^2} \right) \]

For a steel with
\( a = 10 \text{ mm} \),
\( \sigma = 400 \text{ Mpa} \)
\( \sigma_0 = 1500 \text{ Mpa} \)
\( r_p = 0.113 \text{ mm} \)
CRAK LENGTH

CRITICAL CRACK

CORROSION

SPECTRUM CONSTANT R

INITIAL CRACK

DESIGN LIFE

TIME

DAMAGE TOLERANT DESIGN