1. Find the linearization of \( f(x) = \sqrt{x} \) at \( a = 8 \). Use it to give an approximate value for \( \sqrt{7.98} \).

**Solution:** \( f(x) \approx 2 + \frac{1}{12}(x - 8) \), thus \( f(7.98) \approx 2 + \frac{1}{12}(7.98 - 8) = 1.9983 \)

2. Does the function \( f(x) = |x| \) on \([-2, 2]\) satisfy the conditions of the Mean Value Theorem? Why or why not?

**Solution:** No, because \( f'(x) \) does no exist at \( x = 0 \), therefore it is not differentiable on \((-2, 2)\).

3. For \( f(x) = \frac{3 + x}{3 - x} \), find the differential \( dy \).

**Solution:** \( dy = \frac{(3-x) - (3+x)(-1)}{(3-x)^2} \, dx = \frac{6}{(3-x)^2} \, dx \)

4. Determine the vertical and horizontal asymptotes of \( f(x) = \frac{2x^2 + 6x}{x^2 - 9} \).

**Solution:** \( f(x) = \frac{2x(x+3)}{(x+3)(x-3)} \)

\( \lim_{x \to 3} \frac{2x(x+3)}{(x-3)(x+3)} = \infty \) and \( \lim_{x \to -3} \frac{2x(x+3)}{(x-3)(x+3)} = -\infty \), thus we have a vertical asymptote at \( x = 3 \)

\( \lim_{x \to \pm\infty} \frac{2x^2 + 6x}{x^2 - 9} = 2 \), thus we have a horizontal asymptote at \( y = 2 \).

5. Evaluate \( \lim_{x \to 0} \frac{e^{3x} - 1 + 5x}{x^2} \).

**Solution:** \( \lim_{x \to 0} \frac{e^{3x} - 1 + 5x}{x^2} = \lim_{x \to 0} \frac{-5e^{3x} + 5}{2x} = \lim_{x \to 0} \frac{25e^{3x}}{2} = \frac{25}{2} \)

6. Evaluate \( \lim_{x \to \pi/2} \frac{3 \sec x}{2 + \tan x} \).

**Solution:** \( \lim_{x \to \pi/2} \frac{3 \sec x}{2 + \tan x} = \lim_{x \to \pi/2} \frac{3 \sec x \tan x}{\sec^2 x} = \lim_{x \to \pi/2} 3 \sin x = 3 \)

7. Evaluate \( \lim_{x \to \pi/2} (1 - \sin x) \tan x \).

**Solution:** \( \lim_{x \to \pi/2} (1 - \sin x) \tan x = \lim_{x \to \pi/2} (1 - \sin x) \frac{\sin x}{\cos x} = \lim_{x \to \pi/2} \frac{\cos x - 2 \sin x \cos x}{-\sin x} = 0 \)

8. Evaluate \( \lim_{x \to 0} \left( \frac{1}{x^2} - \frac{1}{x^2 \sec x} \right) \).

**Solution:** \( \lim_{x \to 0} \left( \frac{1}{x^2} - \frac{1}{x^2 \sec x} \right) = \lim_{x \to 0} \left( \frac{\sec x - 1}{x^2 \sec x} \right) = \lim_{x \to 0} \left( \frac{\sec x \tan x}{2x \sec x + x^2 \sec x \tan x} \right) \)

\( = \lim_{x \to 0} \left( \frac{\tan x}{2x + x^2 \tan x} \right) = \lim_{x \to 0} \left( \frac{\sec^2 x}{2 + 2x \tan x + x^2 \sec^2 x} \right) = \frac{1}{2} \)

9. Evaluate \( \lim_{x \to 0} \left( 1 - \frac{4}{x} \right)^x \).

**Solution:** \( \lim_{x \to 0} \left( 1 - \frac{4}{x} \right)^x = e^{-4} \)
10. Evaluate $\lim_{x \to 0^+} (1 + x)^{\frac{4}{x}}$

Solution: $\lim_{x \to 0^+} (1 + x)^{\frac{4}{x}} = \exp \left\{ \lim_{x \to 0^+} \frac{4}{x} \ln(1 + x) \right\} = e^4$

11. Find the local and absolute extreme values of the function $f(x) = x - \sqrt{x}$ on $[0, 4]$.

Solution: 

$f'(x) = 1 - \frac{1}{2\sqrt{x}} = \frac{2\sqrt{x} - 1}{2\sqrt{x}}$ so $f'(x) = 0$ when $x = \frac{1}{4}$, so $f(0) = 0$, $f\left(\frac{1}{4}\right) = -\frac{1}{4}$ and $f(4) = 2$.

Therefore absolute maximum occurs at $(4, 2)$ and absolute minimum occurs at $\left(\frac{1}{4}, -\frac{1}{4}\right)$.

12. Given $f'(x) = (x-1)(x+2)(x+4)$, determine the critical points of $f(x)$ and use the second derivative test to determine whether they correspond to local maxima, local minima, or the test is inconclusive.

Solution: $f''(x) = 0$ at $x = 1, -2, -4$. Now $f''(x) = (x-1)(x+2)(x+4) + (x-1)(x+4) + (x+2)(x+4)$ and $f''(1) = 15$, so at $x = 1$ there is a local minimum, $f''(-2) = -6$, so at $x = -2$, there is a local maximum and $f''(-4) = 10$, so at $x = -4$ there is a local minimum.

13. Determine where $f(x) = (1000 - x)^2 + x^2$ is increasing. Use it to decide which is larger $1000^2$ or $998^2 + 2^2$.

Solution: 

$f'(x) = 2(1000 - x)(-1) + 2x = -2000 + 4x$ now $f'(x) = 0$ when $x = 500$ and $f(x)$ is increasing on $(500, \infty)$ and decreasing on $(-\infty, 500)$. Also, $f(0) = 1000^2$ and $f(2) = 998^2 + 2^2$ so since $f$ is decreasing then $f(0) > f(2)$, i.e. $998^2 + 2^2 < 1000^2$.

14. For each function, $f(x) = (x^2 - 1)^3$ and $f(x) = x\sqrt{3 - x}$

Solution: CP at $(0, -1)$, $(1, 0)$, and $(-1, 0)$, $f$ is decreasing on $(-\infty, -1) \cup (-1, 0)$ and increasing on $(0, 1) \cup (1, \infty)$.

Local min at $(0, -1)$, and $f$ is concave up on $(-\infty, -1) \bigcup \left(-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \bigcup (1, \infty)$ and concave down on $\left(-1, -\frac{1}{\sqrt{5}}\right) \bigcup \left(\frac{1}{\sqrt{5}}, 1\right)$.

Solution: Note the domain is $(-\infty, 3)$, CP $(2, 2)$, $(3, 0)$, $f$ is increasing on $(-\infty, 2)$ and decreasing on $(2, 3)$, local max at $(2, 2)$ and the function is concave down on $(-\infty, 3)$

15. Sketch the graph of a function that satisfies all the conditions given below.

Solution: 

16. A metal storage tank with volume $V$ is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal? (The volume of a sphere is $\frac{4}{3}\pi r^3$.)
Solution: Volume: \( V = \pi r^2 h + \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right) \) \( \Rightarrow h = \frac{V}{\pi r^2} - \frac{2r}{3} \), surface area is the sum of the area of the bottom, side, and top. \( S(r) = \pi r^2 + 2\pi rh + \frac{1}{2} (4\pi r) = \frac{5}{3} \pi r^2 - \frac{2V}{r} \), for \( r > 0 \) and \( S'(r) = \frac{10\pi r^3 - 6V}{3r^2} = 0 \) when \( r = \sqrt[3]{\frac{3V}{5\pi}} \), and \( S''(r) = \frac{10\pi}{3} - \frac{4V}{r^3} < 0 \) when \( r = \sqrt[3]{\frac{3V}{5\pi}} \) thus the dimensions that minimize surface area are \( r = \sqrt[3]{\frac{3V}{5\pi}} = h \).

17. A closed box with square base is to be built to house an ant colony. The bottom of the box and all four sides are to be made of material costing \$1/ft^2\ and the top is to be constructed of glass costing \$5/ft^2\ . What are the dimensions of the box of greatest volume that can be constructed for \$72? Verify your answer yields a maximum.

Solution: Cost \( C = x^2 + 4xy + 5x^2 = 6x^2 + 4xy = 72 \), so \( y = \frac{72 - 6x^2}{4x} \) for \( x > 0 \). Now \( V = x^2 y = x^2 \left( \frac{72 - 6x^2}{4x} \right) = 18x - \frac{9}{2} x^3 \) and \( V' = 18 - \frac{9}{2} x^2 = 0 \) when \( x = 2 \). To verify that \( x = 2 \) maximizes volume, use the second derivative test, \( V'' = -9x \) so \( V''(2) = -18 \) thus volume is maximized when \( x = 2, y = 6 \).