Filters

In this laboratory exercise we will examine how to filter an instrumental signal infected with noise so as to improve the signal's signal-to-noise ratio (S/N). We will construct very simple analog \( RC \) filters and measure their attenuation. In one case we will configure the \( RC \) circuit as a Low Pass Filter and in the other as a High Pass Filter.

A filter will remove from our measured signals some undesirable feature or component. Signal filtering can be either analog, as part of the instrumentation, or digital, as part of the instrument's software package. As pointed out by Fountain:

Analog filters have the advantages of being able to operate in real time and require no additional calculations to improve the signal-to-noise ratio. However, they offer only limited control over the measured signal. If the filter is not properly matched, it can also cause phase shifts or unnecessary band broadening to the signal. ..... Digital filtering is a mathematical tool that produces the same smoothing effect on signals as analog filtering with a low pass filter. Digital filtering offers advantages over analog filtering in that it provides for greater control on the output and it does not cause any phase shifts in the resulting signal. Because digital filtering is computationally based, the technique also allows for the use of alternative filters to generate more desirable line 'shapes' in the final signal.

Here we will focus on analog filters. Boxcar or Moving Average Smoothing of digital data, as was performed on the IR Spectrometer data in the lab exercise involving noise measurements, is the equivalent of filtering the data with a low pass filter. High frequency noise is removed from the data.
We will consider a simple RC-circuit that can be used as either a low-pass or high-pass filter. This type of filter is extremely simple and yet will illustrate the basic issues involved in analog filter construction. Low-pass filters do exactly as their name implies, leave intact low frequency signals but attenuate high frequency noise. Exactly the opposite behavior can be expected of a high-pass filter.

We will begin by recalling some basic elements of circuit theory. Let's start by considering a simple Voltage Divider circuit.

\[ V_{\text{in}} \]

\[ R_1 \]

\[ V_{\text{out}} \]

\[ R_2 \]

The input voltage \( V_{\text{in}} \) is to be a simple sinusoidal signal of the form:

\[ V_{\text{in}} = V_o \cos \omega t \]  

(Eq. 1)

where \( \omega \) is the signal's Angular Frequency, determined by its Period (T):

\[ T = \frac{1}{f} = \frac{2\pi}{\omega} \]  

(Eq. 2)

[Diagram of Voltage Divider circuit with input voltage and output voltage labeled.]
We can represent this signal in the complex plane as:

\[ V_{\text{in}} = V_0 e^{j\omega t} \]  
(Eq. 3)

In this representation, our signal is a vector of magnitude \( V_0 \) rotating with an angular frequency of \( \omega \). Circuit analysis becomes greatly simplified using complex algebra and "phasor" diagrams such as the one depicted above. In this representation, passive circuit elements (resistor, capacitor, inductor) behave according to an \( ac \) version of Ohm's Law:

\[ V = I Z \]  
(Eq. 4)

where \( Z \) is the element's Impedance. The impedance \( Z \) (units \( \Omega \)) of the basic passive circuit elements are:

**Resistor (R)**

\[ Z = R \]

**Capacitor (C)**

\[ Z = \frac{1}{j\omega C} \]

**Inductor (L)**

\[ Z = j\omega L \]

Now, the output voltage of the voltage divider can be determined by applying Ohm's Law to the circuit:

\[ V_{\text{in}} = I (Z_{R1} + Z_{R2}) \]  
(Eq. 5)

\[ V_{\text{out}} = I Z_{R1} \]  
(Eq. 6)
Since the current I must be the same thru R₁ and R₂, we have:

\[ \frac{V_{out}}{Z_{R_1}} = \frac{V_{in}}{Z_{R_1} + Z_{R_2}} \]

Thus,

\[ V_{out} = \frac{Z_{R_1}}{Z_{R_1} + Z_{R_2}} V_{in} \]

which reduces to:

\[ V_{out} = \frac{R_1}{R_1 + R_2} V_{in} \]  \hspace{1cm} (Eq. 7)

hence the name "Voltage Divider".

Now consider the following circuit, which will act as Low Pass Filter:

![Circuit Diagram]

Applying the same analysis as above to this circuit, we have:

\[ V_{out} = \frac{Z_C}{Z_R + Z_C} V_{in} \]  \hspace{1cm} (Eq. 8)

\[ = \frac{1}{\frac{1}{j\omega C} + \frac{1}{\omega^2 RC^2}} V_{in} \]

\[ = \frac{1}{\frac{1}{\omega^2 RC^2}} V_{in} \]

Since this circuit acts as a filter, it will attenuate the input voltage at some signal frequencies and not at others. Signal Attenuation A is defined as the ratio of the magnitude of the output to input voltages:

\[ A = \left| \frac{V_{out}}{V_{in}} \right| \]  \hspace{1cm} (Eq. 9)

Here, \(|| \) indicates we are considering only the magnitude of V.
For our circuit then:

\[
A = \frac{1}{\sqrt{1 + (\omega RC)^2}}
\]  
(Eq. 10)

Note that if \( \omega \sim \text{small} \), then \( A \sim 1 \) and the signal is unattenuated. If, on the other hand, \( \omega \sim \text{large} \), then \( A \sim \text{small} \) and the output signal is significantly attenuated. Hence, low frequency signals are "passed" and high frequency signals are "filtered." From (Eq. 10) we can see the Angular Cutoff Frequency, the frequency at which the output voltage drops to \( \frac{1}{\sqrt{2}} \) of the input voltage, is given by:

\[
\omega_c = \frac{1}{RC}
\]  
(Eq. 11)

A signal's attenuation is frequently reported in units of decibles (dB):

\[
A_{\text{db}} = -20 \log \left| \frac{V_{\text{out}}}{V_{\text{in}}} \right|
\]  
(Eq. 12)

For our filter circuit, we have:

\[
A_{\text{db}}
\]

0 0.01 0.1 1.0 10.0 100.0
\( \omega / \omega_c \)

The Phase \( \phi \) of the output signal relative to the input signal is, without derivation, given by:

\[
\phi = \tan^{-1} \left( \frac{-\omega}{\omega_c} \right)
\]  
(Eq. 13)

This can be represented in the phasor and time domain diagrams as:
For our low pass filter, if $\omega << \omega_C$, then $\phi \sim 0^\circ$. However, if $\omega >> \omega_C$, then $\phi \sim -90^\circ$. It is important to note that in many applications, it is undesirable to have the filtered output phase-shifted.

Reconfiguring things slightly, our $RC$-circuit becomes a high pass filter:

For the high pass filter, $\omega_C$ is again $1/RC$. The phase angle is now determined by:

$$\phi = \tan^{-1}\left(\frac{\omega_C}{\omega}\right)$$

(Eq. 14)

As Sprott points out; "More complicated filter circuits can be constructed which have almost any desired attenuation and phase characteristics, although a phase shift inevitably occurs whenever the attenuation varies with frequency. ….. The art of filter design is highly developed, and digital computers are often used to optimize the design of filters for special applications." Here, we are not trying to explore these issues thoroughly. But, are instead trying to develop a feel for what might have to be done in order to filter a noisy instrumental signal.
Procedure

1. Obtain a breadboard and familiarize yourself with its design. (See Appendix)

Voltage Divider

Your laboratory instructor will demonstrate the basic operation of the Function Generator and the Oscilloscope. You will need to be able to adjust the amplitude and frequency of a sinusoidal output from the function generator. You will need to be able to view the voltage at two points in your circuit, using two channels on the oscilloscope, simultaneously. You should be able to measure the magnitude of a sinusoidal signal with the oscilloscope. The following simple circuit will allow you to test your familiarity with the function generator and oscilloscope.

2. Build a simple Voltage Divider circuit. Use resistances of \( R_1 = 1 \, \text{k}\Omega \) and \( R_2 = 1 \, \text{k}\Omega \). (You should measure \( R_1 \) and \( R_2 \) with a Digital Volt Meter and record their accurate values.) Use a Function Generator to supply \( V_{\text{in}} \) with a sinusoidal input. The generator should be set at 10 kHz and 3 V peak-to-peak.
   - Start by building the circuit on the breadboard.
   - Now connect the output of the function generator to the circuit's \( V_{\text{in}} \) and GND.
   - Connect Channel 1 of the oscilloscope to \( V_{\text{in}} \) and GND of the circuit.
   - Likewise, connect Channel 2 of the oscilloscope to \( V_{\text{out}} \) and GND of the circuit.
   - Be sure that all the devices are connected to a common GND.

3. Use the oscilloscope to measure \( |V_{\text{in}}| \) and \( |V_{\text{out}}| \). Be sure to note the signal's frequency.

4. Make the same measurements for the following circuit combinations as well. (Be sure to measure the accurate resistance of the new resistor with the DVM.)

   \[
   \begin{array}{c|c|c}
   \text{Resistance} & \text{Resistance} \\
   \hline
   \text{R}_1 & \text{R}_2 \\
   \hline
   1 \, \text{k}\Omega & 10 \, \text{k}\Omega \\
   10 \, \text{k}\Omega & 1 \, \text{k}\Omega \\
   \end{array}
   \]

Low Pass Filter

5. Set up and measure \( |V_{\text{in}}| \) and \( |V_{\text{out}}| \) of a low-pass RC filter as a function of frequency. Use the 1k\( \Omega \) resistor from above. Determine the capacitance that should be used such that your circuit will have an angular cutoff frequency of \( \omega_c \approx 10 \, \text{kHz} \). Measure \( |V_{\text{in}}| \) and \( |V_{\text{out}}| \) for the following frequencies (\( f \)):
   - 10 Hz
   - 50 Hz
   - 100 Hz
   - 500 Hz
   - 1 kHz
   - 5 kHz
   - …
   - 1000 kHz

At each frequency, make a note of the approximate phase angle. Be sure to record the values of the resistor and capacitor used. You can use an input signal of 3 V peak-to-peak.
High Pass Filter

6. Set up and measure $|V_{in}|$ and $|V_{out}|$ of a high-pass RC filter as a function of frequency. Your circuit should have an angular cutoff frequency of $\omega_c \sim 10$ kHz. (You can use the same resistor and capacitor as above.)

Band Pass Filter

7. Build a circuit that cascades a low-pass RC filter and a high-pass RC filter such that the circuit will act as a Band Pass Filter. It should cut off frequencies below 1 kHz and above 10 kHz. Start by sketching the circuit to be constructed. Then build the circuit and measure $|V_{in}|$ and $|V_{out}|$ as a function of frequency.
Data Analysis

1. For each Voltage Divider circuit, determine:

\[
\frac{V_{\text{out}}}{V_{\text{in}}}
\]

using your measured voltages. Compare this with the expected result.

2. For each filter circuit:

i) Calculate \( A_{\text{db}} \) vs frequency for each frequency for which measurements were taken.

ii) Prepare a plot of \( A_{\text{db}} \) vs \( \log(f) \). Mark the expected \( f_c \) value on the plot; \( f_c = \omega_c / 2\pi \).

(Note: Positive \( A_{\text{db}} \) values are plotted "southward".)

iii) Comment on the behavior of the phase angle \( \phi \) as the frequency changes.

You should submit a Memo Style report of the above findings. Include all plots requested.
Appendix

Resistor Color Code taken from [http://www.token.com.tw/resistor/resistor-color-code.htm](http://www.token.com.tw/resistor/resistor-color-code.htm). They have a nice discussion on how to read the color code as well.

# TOKEN RESISTOR COLOR CODE

<table>
<thead>
<tr>
<th>COLOR</th>
<th>1ST BAND</th>
<th>2ND BAND</th>
<th>3RD BAND</th>
<th>MULTIPLIER</th>
<th>TOLERANCE</th>
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<tr>
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<td>1</td>
<td>10</td>
<td>± 1%</td>
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<tr>
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<td>2</td>
<td>2</td>
<td>2</td>
<td>100</td>
<td>± 2%</td>
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<td>3</td>
<td>3</td>
<td>1K</td>
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<td></td>
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<td>± 20%</td>
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Breadboard Configuration

The following schematic is taken from http://tymkrs.tumblr.com/post/638624174/how-to-use-a-breadboard.

The “bus strips” run the entire length of the breadboard and are for connecting to ground and power.

On some breadboards the bus strip only runs halfway - you need to bridge this gap if you want to use the entire strip.

The notch in the middle is for adding IC chips.

The “terminal strips” run in columns of 5 and are for adding components to the board.
References


