Problem Set 2

Problems

1. Use the Maxwellian Distribution of Energies to explicitly show the average molecular energy is:

\[ \langle \varepsilon \rangle = \frac{3}{2} kT \]

2. If heavier gas molecules move more slowly than light gas molecules, why is the average kinetic energy independent of mass?

3. The fraction of molecules in a gas with kinetic energy greater than \( \varepsilon^* \) is given by the following integral:

\[ f(\varepsilon^*) = 2\pi \left( \frac{1}{nkT} \right)^{3/2} \int_{\varepsilon^*}^{\infty} \sqrt{\varepsilon} \ e^{-\varepsilon/kT} \, d\varepsilon \]

Show that this integral is given by:

\[ f(\varepsilon^*) = \frac{2}{\sqrt{\pi}} a \ e^{-a^2} + \text{erfc}(a) \]

where:

\[ a = \varepsilon^*/kT \]

and \( \text{erfc}(a) = 1 - \text{erf}(a) \) is the complementary error function.

Consider making the following change of variable; let:

\[ \varepsilon = kTx^2 \]

and then integrate by parts. (Note, you are not deriving \( f(\varepsilon^*) \), merely carrying out the transformation.)

4. What fraction of molecules in gas have energies greater than \( kT \), \( 2kT \), \( 5kT \), \( 10kT \)?
5. A velocity selector like that in the *Miller & Kusch* paper is constructed. The distance between the discs is 5cm. The displacement $\theta$ of the slots in the discs is 5°. Calculate the rotational frequency required to allow Na atoms having a Speed equal to the most-probable-speed at 1000K to be detected. The Na beam passes through the slots at a distance of 1cm from the center.

6. For O$_2$ at 25°C and 1.00 atm, estimate the number of collisions per molecule per unit time. Take the bond distance for the O$_2$ molecule as 1.2Å. Also calculate the total number of collisions per unit time per unit volume.

**Additional Problems for 527 Students**

9. The concentration of CO$_2$ in the atmosphere is approximately 0.04 mole percent. Calculate the Rate at which CO$_2$ molecules strike 100 cm$^2$ of leaf on a tree at 300K. The overall reaction for photosynthesis can be taken as:

$$6 \text{CO}_2 + 6 \text{H}_2\text{O} \rightarrow \text{C}_6\text{H}_{12}\text{O}_6 + 6 \text{O}_2$$

The *Malus pumila* (Common Apple Tree) is found to consume 20mg of CO$_2$ per hour per 100 cm$^2$ of leaf area. What is the fraction of the CO$_2$ molecules that strike the leaf and are eventually converted to C$_6$H$_{12}$O$_6$?