1. Find the volume of the solid generated by revolving about the $x$-axis the wedge-like plane region bounded by the lines $y = \sec x, \ y = \sqrt{2}, \ -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$. \\
\[ \text{Washer Method} \]
\[ V = \pi \int_{-\pi/4}^{\pi/4} [((\sqrt{2})^2 - (\sec x)^2)] \, dx = \pi \int_{-\pi/4}^{\pi/4} (2 - \sec^2 x) \, dx \]
\[ = \pi \left( 2x - \tan x \right) \bigg|_{-\pi/4}^{\pi/4} = 2\pi \left( \frac{\pi}{4} - \frac{\pi}{4} \right) = \pi \left( \pi/2 \right) \]
\[ = 2\pi \left( \frac{\pi}{4} \right) = \pi \frac{\pi}{2} = \frac{\pi^2}{2} \]

2. Find the volume of the solid defined as follows. The base of the solid is the circular disk in the $xy$-plane described by $x^2 + y^2 = 1$, and the cross-sections by planes perpendicular to the $x$-axis are squares. Hint: the cross-sections are between $x = -1$ and $x = 1$, and the side of the cross-section square in $xy$-plane at point $(x, 0)$ is between the curves $y = \sqrt{1-x^2}$ and $y = -\sqrt{1-x^2}$.

\[ V = \int A(x) \, dx, \quad A(x) \text{- area of cross section} \]
\[ A(x) = \pi; \quad dA = (\sqrt{1-x^2} - (-\sqrt{1-x^2}))^2 = 4(1-x^2) \]
\[ V = \int_1^0 4(1-x^2) \, dx = 4 \left( x - \frac{x^3}{3} \right) \bigg|_1^0 = 4 \left( -\frac{1}{3} \right) - 4 \left( -1 + \frac{1}{3} \right) = 8 \left( -\frac{1}{3} \right) = \frac{16}{3} \]

3. Find the volume of the solid generated by revolving about the line $x = 1$ the region in $xy$-plane bounded by the curves $y = 2x - x^2$ and $y = x$.

\[ y = 2x - x^2 = 0 \Rightarrow x = 0, x = 2, \quad y(1) = 2 \cdot 1 - 1^2 = 1 \]

\[ \text{Shells Method} \]
\[ V = 2\pi \int_0^1 (1-x)(2x-x^2-x) \, dx = 2\pi \int_0^1 (1-x)(x-x^2) \, dx \]
\[ R = 1-x \]
\[ = 2\pi \int_0^1 (x-x^2-x^2+x^3) \, dx = 2\pi \left( \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \right) \bigg|_0^1 \]
\[ = 2\pi \left( \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \right) = 2\pi \frac{6-8+3}{12} = \frac{\pi}{6} \]