1. Consider a one-dimensional collision between two bodies of masses $m_1$ and $m_2$ and initial velocities $v_{1i}$ and $v_{2i}$. The final velocities are $v_{1f}$ and $v_{2f}$. If the collision is elastic, both momentum and kinetic energy are conserved. The conservation equations can be solved to show that the final velocities are

$$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i},$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{(m_2 - m_1)}{(m_2 + m_1)} v_{2i}.$$

For the special case when $m_1 = m_2 = m$, the above equations give that $v_{2f} = v_{1i}$ and $v_{1f} = v_{2i}$. This implies that the relative approach velocity before the collision is the same as the relative separation velocity after the collision.

(a) Use the above results to solve Problem 1.13 of the text, concerning the dissociation of a two-body oscillator. The two particles in the oscillator are both of mass $m$ and one of the particles is struck head-on by a third particle also of mass $m$ and incoming speed $u$. Dissociation is assumed to occur when the relative displacement of the particles in the two-body system has an amplitude $x_0 = \xi_{\text{max}}$. (Answer: The oscillator dissociates when $u = \sqrt{2K/m} \xi_{\text{max}} = \omega_0 \xi_{\text{max}}$, where $\omega_0$ is the natural oscillation frequency of the two-body oscillator.)

(b) What fraction of the initial kinetic energy goes into translation of the center of mass of the two-body system, and what fraction goes into the oscillations?

(c) Evaluate $u$ numerically for $\xi_{\text{max}} = 1 \text{Å}$ and $f_0 = 10^{13} \text{Hz}$, as in Problem 1.17.

(d) Compare with the rms speed of O$_2$ molecules at room temperature, obtained by equating the kinetic energy of the molecule to $(3/2)kT$. Will O$_2$ dissociate at room temperature due to collisions with other O$_2$ molecules?

2. Determine the restoring force $F$ for the transverse mass-spring oscillator of Problem 1.18 of the text, as a function of the transverse displacement $\xi$. As discussed in the text problem, this is an example of an oscillator where the restoring force is not linearly proportional to the displacement. Do the problem in two ways:

(a) by drawing a free-body diagram of the forces acting on the mass $m$ and by determining the resultant of the forces, and

(b) by determining the potential energy $V(\xi)$ and taking the negative gradient of $V$.

Answer: $F = -2K\xi \left(1 - \frac{L}{\sqrt{L^2 + \xi^2}}\right)$

NOTE: Consider the system to consist of two separate springs, each having a spring constant $K$ and relaxed length $L$ (rather than one spring of length $2L$ and spring constant $K$, as the text problem states).

3. For the transverse mass-spring oscillator of the previous HW set, the restoring force was found to be $F = -2K\xi(1 - L/\sqrt{L^2 + \xi^2})$. Often, the restoring force is linearly proportional to displacement $\xi$ when the displacement is small. In this problem we determine if this is the case for the transverse mass-spring system. (a) Determine how $F$ varies with $\xi$ for small values of displacement ($\xi \ll L$). To do this, note that

$$\frac{L}{\sqrt{L^2 + \xi^2}} = \frac{1}{\sqrt{1 + (\xi/L)^2}},$$

where $\xi/L$ will be much less than unity. Then use the binomial series expansions for $(1 + x)^{1/2}$ and for $1/(1 + x)$, where $x$ is a small quantity. (Answer: $F = -(K/L^2)\xi^2$ for $\xi/L \ll 1$.)

(b) Use MATLAB to plot the restoring force and potential energy as a function of displacement $\xi$. Let the spring constant $K = 20$ N/m and the length of each spring be $L = 0.3$ m. For the restoring force plot, overlay the small-displacement approximation for $F$ as a dashed line to show that it indeed corresponds to the force when $\xi/L$ is small.

4. The Morse potential, discussed in Display 1.8 of Ingard’s text, empirically describes the interaction potential between two atoms or molecules in a solid. Among other things, the Morse potential shows why it is that solids expand slightly when they are heated (thermal expansion). It is interesting to read Ingard’s discussion of this and other aspects of the Morse potential in the text accompanying Display 1.8, in the subsection titled ‘Molecular vibrations and related matters’ beginning on page 21.

Ingard gives the Morse potential in Equation 1 of Display 1.8. Letting $x' = \xi = x - x_0$ be the displacement from the potential minimum, and $E_0$ be Ingard’s parameter $D$, the Morse potential is given by

$$V(x') = E_0[e^{-2b x'} - 2e^{-bx'}].$$

Starting with the Morse potential, derive the remaining results in the display. Namely, show

(a) that the energy of the potential minimum, $V(x_0) = -E_0$,

(b) that the potential energy at $x = \infty$ is zero,

(c) the expression for the restoring force $F$ as a function of $x'$ (or $\xi$).

(d) Also show that, for small displacements $x'$, the restoring force is linearly proportional to the displacement, with the constant of proportionality (i.e., the effective spring constant) being given by $K = 2b^2 E_0$. (To do this, use the first two terms of the polynomial series expansion for $e^{-x}$.)

(e) Determine the expression for the natural oscillation frequency $\omega_0$ for small displacements.

(f) Qualitatively sketch $V(x)$ and $F(x)$ versus $x$, labeling the minimum potential value.

(g) From the shape of the potential energy curve, why do solids exhibit thermal expansion when they are heated?