



Modeling Security through Copulas

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March 7, 2005



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7 March 2005

Overview

- Extreme Value Theory
- Copulas
 - Financial Applications
 - Security Modeling
- Current Applications
- Questions

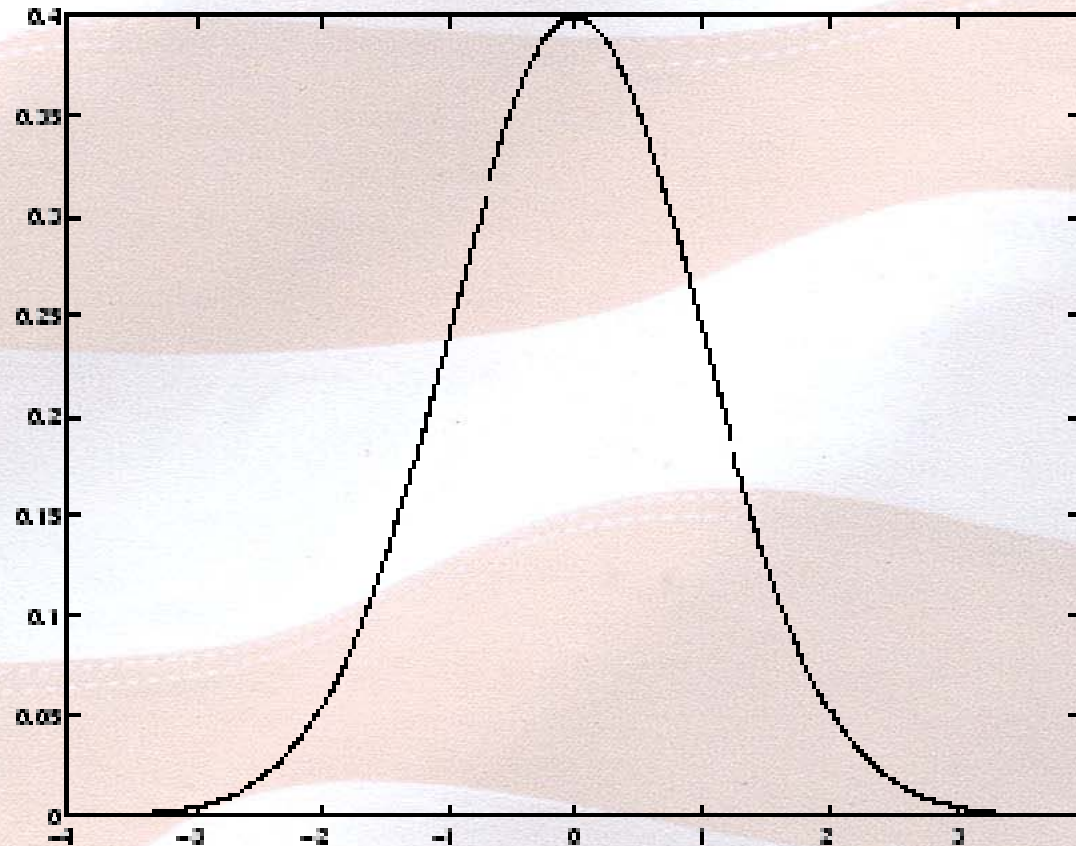


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Extreme Value Theory

- **Extreme value theory** is a branch of statistics dealing with the extreme deviations from the median of probability distributions.
- Uses
 - Rare events
 - Financial Markets
 - Engineering

Standard Normal Distribution



Outliers

- Extreme points that do not fit within 3σ of the mean of the distribution
 - .26% of Z values in standard normal
 - $< 1\%$ in most true normal distributions



Why do we care?

- Outliers show exceptions to standard behavior
- Immediately signal something in the system has changed



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Copulas

- functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution function
- Dependence function of random variables



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Copula Transformations

- Transform uniform random variables into a copula by using marginal distribution integration functions

$$C_{\rho}^{Ga}(x, y) = \int_{-\infty}^{\Phi^{-1}(x)} \int_{-\infty}^{\Phi^{-1}(y)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left\{\frac{-(s^2 - 2\rho st + t^2)}{2(1-\rho^2)}\right\} ds dt,$$

Financial Markets

- Copulas represent the dependence of extreme events based on other extreme events
 - Market fluctuations
 - International currencies

Risk Management

- Risks of the marketplace
- Operations Theory
- Economic capital adequacy



Computer Security

- Network Traffic Monitoring
 - Packet flow
- Usage statistics (MIS / IT applications)
- Focused communication patterns

Future Work

- Analyze copulas and run test implementation on financial data
- Transform problem into cyber security framework
- Build models of security framework
- See if added copulas improve overall security in information assurance

Questions



Sources

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<http://www.math.ethz.ch/~mcneil/ftp/tCopula.pdf>
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Sources

- Nelsen, Roger B. **Introduction to Copulas**. Lecture Notes in Statistics Series. Springer-Verlag. Ann Arbor, MI. 1999.