# 1 On the Theoretical Basis of Photons

Pharis E. Williams
20 August, 2000

Abstract: Weyl's quantum principle provides the logical connection, for which Einstein searched in vein, between a photon's wave and particle properties.

## 1.1 Light Quanta

The concept of a photon started with Einstein's light quanta[1]. The concept has been the subject of many articles since 1905. The name photon was ørst introduced by Lewis 21 years later[2]. However, both Planck's [3] and Einstein's derivations of the famous relation between energy and frequency, $\epsilon = h\nu$, came from studies of radiation in thermal equilibrium with a system described by statistical thermodynamics. Planck quantized the equilibrium energy U of an oscillator while Einstein quantized the entropy density per unit volume. In 1917 Einstein wrote, 'The properties of elementary processes required by [his momentum Æuctuation relation] make it seem almost inevitable to formulate a truly quantized theory of radiation.[4]' Einstein was not, and never would be satisøed with his, and others, inability to obtain such a theory. In 1924, after the experimental evidence of the Compton Eœect provided proof of the quantization of light, he wrote, 'There are therefore now two theories of light, both indispensable, and¯as one must admit today despite twenty years of tremendous eœort on the part of theoretical physicists¯without any logical connection[5].'

Weyl's quantum principle may be used to derive Maxwell's electromagnetic øeld equations[6]. These, in turn, may be used to derive the electromagnetic wave equations. At the same time the quantum principle requires that the gauge potentials be quantized since $\int N_j \phi_j dx^j = 2\pi i N$ where $i = \sqrt{-1}$ and there is on summation over the j's. The radial electrostatic dependence may be investigated by considering $N_o$ to be nonzero and $N_x$=$N_y$=$N_z$=0. The concept of a photon, as a particle, is one that is electrically neutral. The wave description allows the discussion of polarization such that an electromagnetic wave traveling along the x axis may have its electric øeld directed along the y axis. There, consider $N_o$=$N_x$=$N_z$=0. The quantum principle requires that the y component of the gauge (vector) potential to be quantized, as $\phi_y = NB\cos 2\pi\left(\frac{x}{\lambda} - \nu t\right)$ where the dependence upon x and t was chosen to be sure that the electric øeld, given by $\xi\left(x,t\right) = \frac{\partial\phi_y(x,t)}{\partial(ct)} = \frac{NB\nu}{c^2}\sin 2\pi\left(\frac{x}{\lambda} - \nu t\right)$, satisøes the wave equations. This expression may be used to ønd the average value of the Poynting vector, or $I = \left(\frac{1}{\mu_o}\right)\langle\xi^2\rangle$. The average value of the square of electric øeld is given by $\langle\xi^2\left(x,t\right)\rangle = \frac{N^2B^2\nu^2}{c^3}\int_{t=0}^{t=\frac{1}{\nu}}\sin^2 2\pi\left(\frac{x}{\lambda} - \nu t\right)dt = \frac{N^2B^2\nu}{2c^2}$.

Therefore, $I = \left(\frac{N^2B^2}{2\mu_o c^3}\right)\nu$. Now a quantum of light for which N=1 would have an energy Æow of $I = h\nu$ when $B = \sqrt{2\mu_o hc^3}$. Einstein's energy relation, for a single light quantum passing through a unit area, is then $\epsilon = h\nu$.

## 1.2 Conclusions

A photon may be deøned as a gauge vector potential with the quantum number set to unity and which satisøes the wave equation. Further, the light quanta comes from Weyl's quantum principle which also provides the basis for radiation and needs no connection to statistics or thermodynamics. Weyl's quantum principle provides the logical connection that Einstein was trying to ønd.

References:

1..A. Einstein, Annalen der Physik, 17, 132 (1905).

2..G. Lewis, ıThe Conservation of Photons,ȷ Nature, 118, 874 (1926).

3..M. Planck, Verh. Deutsch. Phy. Ges. 2, 237 (1900).

4..A. Einstein, Phys. Zeitschr. 18, 121 (1917).

5..A. Einstein, Berliner Tageblatt, April 20, 1924.

6..H. Weyl, Zeit. F. Phys. 330, 56 (1929).