

# Thermodynamics/speed of light/Equations of motion

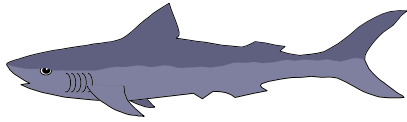
*Logs in the road/wheelbarrow velocity/how do I get there from here?*

Presented by  
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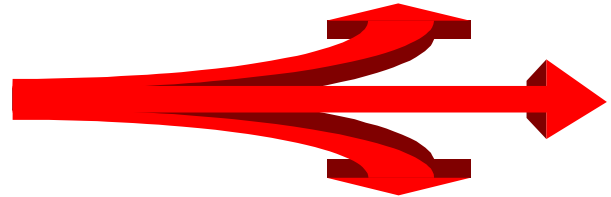
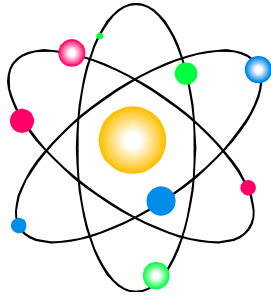
8 February 2001



*"Of all the communities available to us there is not one I would want to devote myself to, except for the society of the true searchers, which has very few living members at any time."*



# From a fishing hole to physics



**Why not Newtonian Mechanics?**



**Why not relativistic mechanics?**

**Why not quantum mechanics?**



**Why thermodynamics?**

# Because we want it that way!

“The conservation laws are in a sense not laws at all, but postulates which we insist must hold in any physical theory... We prefer always to look for quantities which are conserved, and agree to apply the names ‘total energy,’ ‘total momentum,’ ‘total angular momentum’ only to such quantities. The conservation of these quantities is then not a physical fact, but a consequence of our determination to define them in this way.”

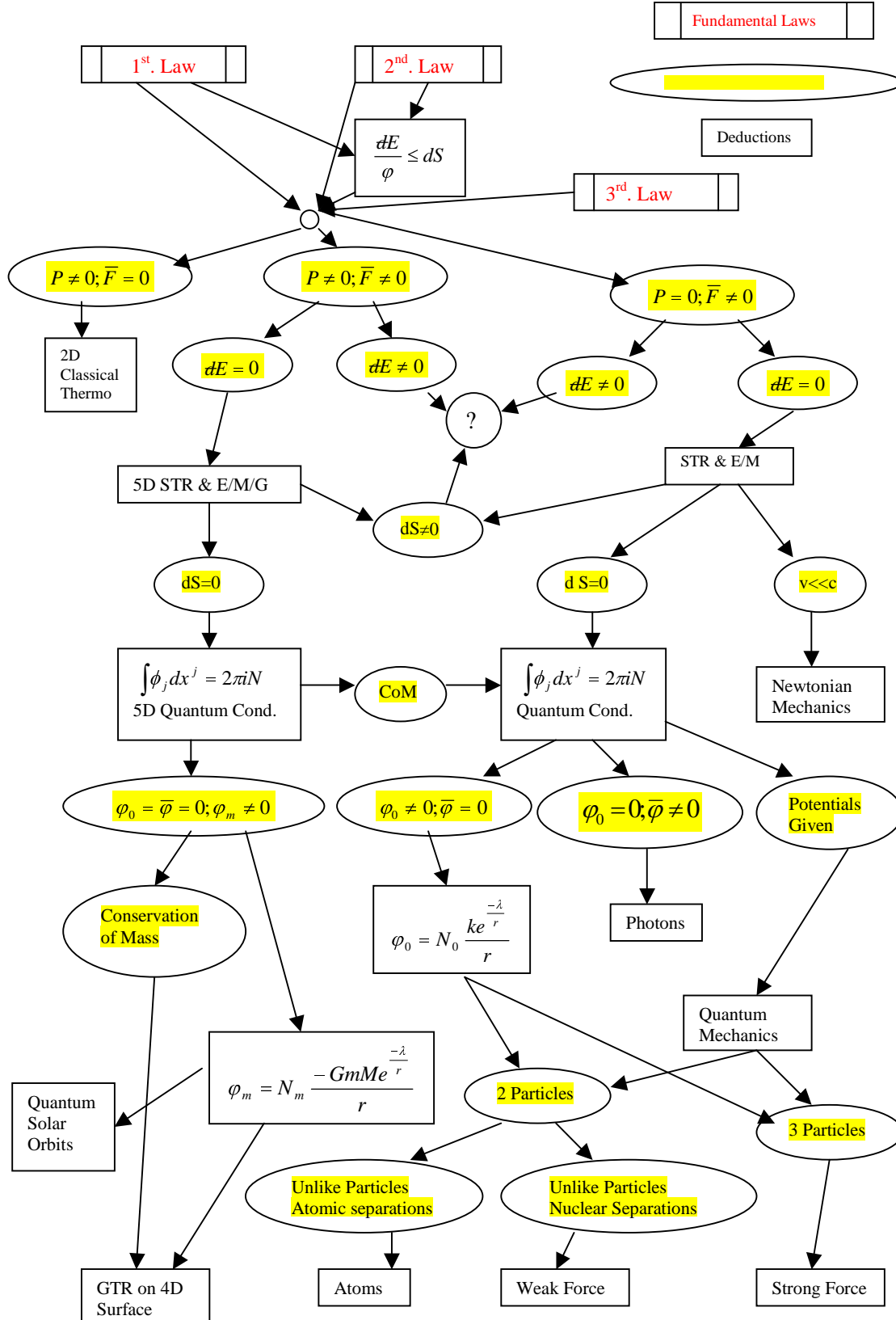
Symon, “Mechanics”

# The Lagrangian tells us so

“Conservation laws can be regarded as consequences of symmetries exhibited by the mechanical systems to which they apply; that is, they are consequences of the fact the Lagrangian function  $L$ , which determines the equations of motion, is independent of time and of the position and orientation of the entire system in space.

Symon, “Mechanics”

# Dynamic Theory Logic Flow



# What are we throwing away?

First law of Thermodynamics  
(Conservation of Energy)

$$0 = \Delta KE + W_{cons} + W_{friction} + W_{non-cons} + \Delta U_{internal} + \bullet E$$

Path dependent things

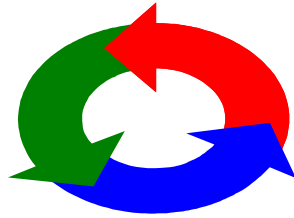
$$W_{friction}; W_{non-cons}; \bullet E$$

**Toss these suckers!**  
**Also, hold the internal energy constant.**

Classical Conservation Energy

$$0 = \Delta KE + W_{cons}$$

**Conservation  
Laws**



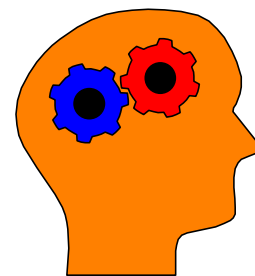
**Equations of  
Motion**

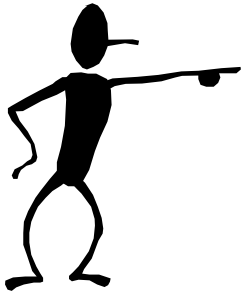
***Energy makes Force!***

***Force does not make energy!***



***So what?***





# First Law

- $E = dU - F_x dx - F_y dy - F_z dz = dU - F dx$

**Rewrite**

- $E = dKE(v) - F(v, x) dx$

**Or**

- $\frac{E}{dt} = \frac{\partial KE}{\partial v} a - F(v, x) v$

- $\frac{E}{dt} = m v a - F(v, x) v$

**If •  $E = 0$ , then**

$$m a = F(v, x); v \neq 0$$

**Is this the equation of motion?**

**You can't get SOMETHING for nothing!!**



## **Second Law**

In the neighborhood (however close) of any state of a system of any number of dynamic coordinates, there exist states that cannot be reached by reversible E-conservative ( $\bullet E=0$ ) processes (paths).

This guarantees there exists an integrating denominator for the First Law

$$\frac{\bullet E}{\phi(v)} = f(\sigma) d\sigma \equiv dS$$

$dS$  is a path independent, total differential.

# Result of the Second Law

There exists an integrating factor for the First Law such that:

$$\frac{\bullet E_R}{\phi(\dot{q})} = f(\sigma) d\sigma \equiv dS \quad \text{is a perfect differential}$$

**Unique Velocity**  $\Delta E = \phi(\dot{q}) \int_{\sigma_2}^{\sigma_1} f(\sigma) d\sigma$

for constant velocity

$$\Delta E = \phi(\dot{q}) \left[ \frac{\Delta E_3}{\phi(\dot{q}_e)} \right] \quad (\text{both } \dot{q} \text{ and } \dot{q}_3 \text{ are constant})$$

## **Definition:**

The unique velocity,  $c$ , is that constant velocity at which a system may go from one  $E$ -conservative curve to another without exchange of energy.

## **Lorentz Transformations**

Required by the First and Second Laws

# Equations of Motion

1. Can be obtained from a metric and variational principle
2. Second Law provides a metric in the “stability” conditions
3. Second Law provides a variational principle in the generalized entropy principle
4. Because the Laws provide the metric they should also specify the required type of geometry

# Equilibrium and Stability

## Equilibrium Condition

Free Energy:  $\Delta A = \Delta E - \Delta U + F_i \Delta q^i$

Then Clausius' Inequality Requires:

$$\delta A \leq 0 \quad (\text{Minimum Free Energy Principle})$$

Also, if  $\bullet E = 0$  (Isolated System)

$$\text{Then, } \delta S \geq 0 \quad (\text{Maximum Entropy Principle})$$

## Stability Condition

$$\delta U - F_i \delta q^i - \phi \delta S > 0$$

Leads to the Quadratic Form

$$h_{jk} dq^j dq^k; j, k = 0, 1, 2, \dots, n$$

Where  $q^0 = \frac{S}{f_0}$  and  $h_{jk} = \frac{\partial^2 U}{\partial q^j \partial q^k}$

# Space-Time Manifold

Parameterize  $(ds)^2 = h_{jk} dq^j dq^k$

By Choosing  $ds = c dt$

Where  $c$  is the unique velocity, and  $t$  is the time

Therefore  $(c dt)^2 = h_{jk} dq^j dq^k$

For •  $E = 0$  we have the entropy principle

and  $(dq^0)^2 = (f) \{c^2 dt^2 + 2h'_{0\alpha} dq^\alpha dq^\beta\}$

Where  $f$  and  $h'_{0\alpha}$  are functions of the  $h_{jk}$  and  
• , • = 1, 2, ..., n

Then  $(dq^0)^2 = f (dó)^2$  or  $(dq^0)^2 = g_{jk} dx^j dx^k$ ;

$j, k = 0, 1, 2, \dots, n$   $(dó)^2 = \hat{g}_{jk} dx^j dx^k$

Where  $x^0 = ct$  and  $q^\alpha = x^\alpha$

# Geometry

What Type Is Required By The Laws?

$dq^0$  must be an exact differential and  $dq^0 = q_j^0 dx^j$

Where:  $x^0 = ct; x^\alpha = q^\alpha; \alpha = 1, 2, \dots, n$

Then:  $q_{j,k}^0 - q_{k,j}^0 = 0$

Definition: Parallel Displacement of a Vector

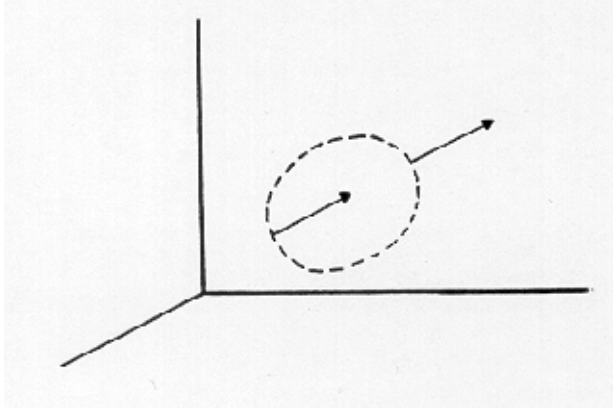
$d\xi_j = \Gamma_{jl}^k dx^l \xi_k$  leads to a requirement that

$(dq^0)^2$  is a Riemannian Space

and  $(d\sigma)^2$  is a Weyl Space

The Gauge Function,  $f$ , is a Geometrical  
Integrating Factor

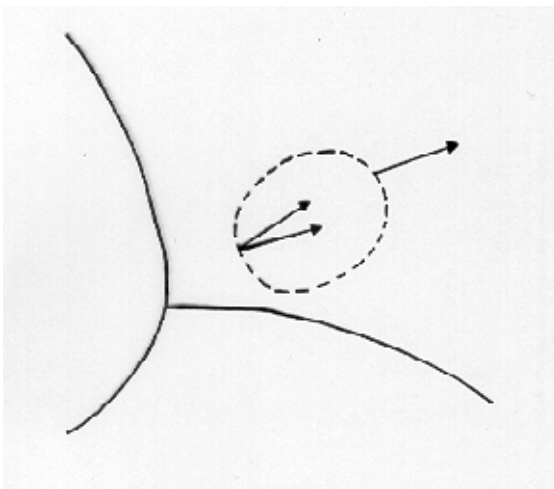
# Types of Geometry



## Euclidean

$$d(\xi_i \xi^i) = 0$$

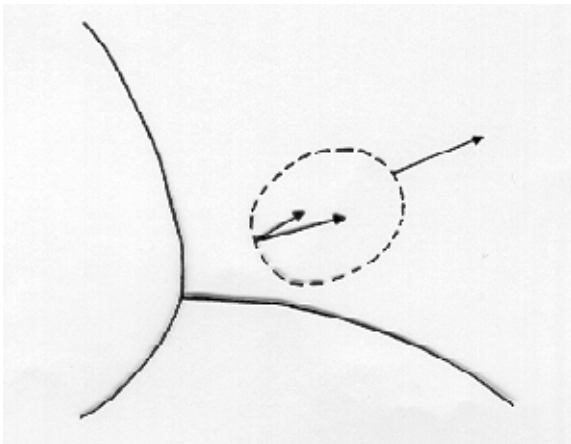
$$d\xi_i = 0$$



## Riemannian

$$d(\xi_i \xi^i) = 0$$

$$d\xi_i = \Gamma_{ij}^r dx^j \xi_r$$



## Weyl

$$d(\xi_i \xi^i) = \phi_k dx^k (\xi_j \xi^j)$$

$$d\xi_i = \Gamma_{ij}^r dx^j \xi_r$$

# Equations of Motion

For isolated systems, •  $E=0$

$$\delta \int dS = \delta \int dq^0 = \delta \int (dq^0)^2$$

Since the Entropy Space is Riemannian  
These equations of motion are Einstein's  
relativistic equations.



Conservation of Energy  
**DOES** require  
Equations of Motions!

**Plus, something extra!**

## Conservation of Energy:

$$\bullet E = \frac{\partial U}{\partial v} dv + \left[ \frac{\partial U}{\partial x} - F \right] dx$$

## Differential of the Mechanical Entropy

$$dS = \frac{\frac{\partial U}{\partial v}}{\phi(v)} dv + \frac{\left[ \frac{\partial U}{\partial x} - F \right]}{\phi(v)} dx$$

but

$$\frac{\partial^2 U}{\partial v \partial x} = \frac{\partial^2 U}{\partial x \partial v} \quad \text{and} \quad \frac{\partial^2 S}{\partial v \partial x} = \frac{\partial^2 S}{\partial x \partial v}$$

Then if  $U=U(v)$

$$\bullet E = \frac{dU}{dv} dv - \phi(v) F(x) dx$$

$$dS = \frac{\frac{dU}{dv}}{\phi(v)} dv - F(x) dx$$

# Energy Concepts

## System Energy

$$U = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Rest Energy

$$RE = m_o c^2$$

## Kinetic Energy

$$KE = U - RE = m_o c^2 \left[ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right]$$

# Energy Concepts (Continued)

## Mechanical Entropy

$$S = \frac{\frac{1}{2} m_o v^2}{\left[ 1 - \frac{v^2}{c^2} \right]} + V(x)$$

where

$$V(x) \equiv - \int F(x) dx$$

## Mechanical Enthalpy

$$H \equiv U + \sqrt{1 - \frac{v^2}{c^2}} V(x)$$

# Energy Concepts (Continued)

## Mechanical Helmholtz's Free Energy

$$\begin{aligned} A &\equiv U - \sqrt{1 - \frac{v^2}{c^2}} S \\ &= \frac{m_o \left[ 1 - \frac{3v^2}{2c^2} \right]}{\left[ 1 - \frac{v^2}{c^2} \right]^{\frac{3}{2}}} - \sqrt{1 - \frac{v^2}{c^2}} V(x) \end{aligned}$$

## Mechanical Gibb's Free Energy

$$\begin{aligned} G &\equiv H - \sqrt{1 - \frac{v^2}{c^2}} S \\ &= \frac{\frac{1}{2} m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}$$