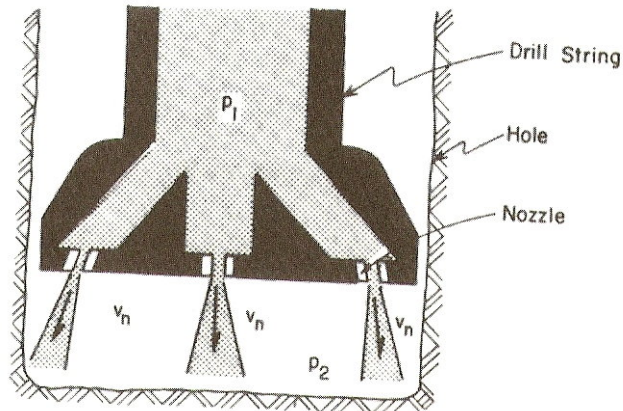


## Drill Bit Hydraulics



### Assumptions

- 1) Change of pressure due to elevation is negligible.
- 2) Velocity upstream is negligible compared to nozzles.
- 3) Pressure due to friction is negligible.

$$\Delta P_B - 8.075E - 4\rho v_n^2 = 0$$

$\Delta P_B$  Pressure drop across bit,  $v_n$  nozzle velocity

Solving for nozzle velocity

$$v_n = \sqrt{\frac{\Delta P_B}{8.074E - 4\rho}}$$

In the field it has been shown that velocity predicted by this equation is off. So it has been modified,

$$v_n = C_d \sqrt{\frac{\Delta P_B}{8.074E - 4\rho}}$$

the recommended value for  $C_d$  is .95.

If 3 nozzles are present

$$v = \frac{q_1}{A_1} = \frac{q_2}{A_2} = \frac{q_3}{A_3} \quad \text{the velocity is equal in all the jets.}$$

$$q = q_1 + q_2 + q_3 = v_n A_1 + v_n A_2 + v_n A_3$$

That gives us

$$v_n = \frac{q}{A_t} \quad \text{In field units} \quad v_n = \frac{q}{3.117 A_t}$$

q in gpm,  $A_t$  in inches<sup>2</sup>,  $v_n$  in ft/sec

solving for the pressure drop

$$\Delta P_B = \frac{8.311E - 5 \rho q^2}{C_d^2 A_t^2}$$

$\rho$  is #/gal

Flow Exponent  $\alpha$

It can be deduced that

$$P_f = CQ^\alpha \quad C \text{ is a constant}$$

$$\log P_f = \log C + \alpha \log Q$$

So the log log plot of this equation is a straight line with a slope of  $\alpha$ .

$\alpha$  can found if two  $P_f$  and  $Q$  are known, this can be achieved by measuring the standpipe or surface pressure for 2 pumping rates.  $P_s = P_f + P_B$  so by using the above equation  $P_B$  can be calculated and subtracted from  $P_s$  to find  $P_f$ .

$$P_f = P_s - \frac{8.311E - 5 \rho q^2}{C_d^2 A_t^2} \quad \text{after finding } P_f, \alpha \text{ can be found by}$$

$$\alpha = \frac{\log\left(\frac{P_{f2}}{P_{f1}}\right)}{\log\left(\frac{Q_2}{Q_1}\right)}$$

Maximum Drill Bit Hydraulic Horsepower Criterion assumes that optimum hole cleaning is achieved if the hydraulic horsepower across the bit is maximized with respect to the flow rate Q.

$$H_{HB} = P_B Q$$

Sub in

$$P_B = P_s - CQ^\alpha$$

$$H_{HB} = P_s Q - CQ^{\alpha+1}$$

Take the first derivative of H with respect to Q set the result to 0.

$$\frac{dH_{HB}}{dQ} = P_s - (\alpha + 1)CQ^\alpha = 0$$

$$P_f = CQ^\alpha \quad P_s - (\alpha + 1)P_f = 0 \quad \text{or} \quad P_f = \frac{1}{\alpha + 1} P_s$$

this is the root that makes  $H_{HB}$  a maximum.

Hence the optimum bit hydraulics will be achieved if friction pressure loss in the system is maintained at an optimum value of

$$P_{f_{opt}} = \frac{1}{\alpha + 1} P_{s_{max}}$$

across the nozzles

$$P_{B_{opt}} = P_{s_{max}} - P_{f_{opt}} = \frac{\alpha}{\alpha + 1} P_{s_{max}}$$

Calculate or measure a  $P_{fqa}$  @ some  $Q_a$  then knowing  $P_{fopt}$  a  $Q_{opt}$  can be calculated by

$$Q_{opt} = Q_a \text{ anti log } \left[ \frac{1}{\alpha} \log \left( \frac{P_{fopt}}{P_{fqa}} \right) \right]$$

With  $Q_{opt}$  known the  $P_{Bopt}$  can be rewritten

$$P_{Bopt} = \frac{8.3E - 5\rho Q_{opt}}{A_{topt}^2 C_d^2}$$

solve for  $A_{topt}$

$$A_{topt} = \sqrt{\frac{8.3E - 5\rho Q_{opt}}{C_d^2 P_{Bopt}}}$$

if all nozzles are the same size  $A_{topt} = \frac{\pi}{4} n d_{nopt}^2$   $n$  is the number of nozzles

solve for  $d_{nopt}$

$$d_{nopt} = \sqrt{\frac{A_{topt}}{n\pi}}$$

Example:

DP 41/2" 20#/ft, Collars 7" 120.3#/ft 1000'

Mud  $\theta_{300}$  21,  $\theta_{600}$  29,  $\rho$  15.5 #/gal

Pump  $P_{\max}$  5440 psi  $H_{HP}$  1600hp 80%

TD 12,000'  $V_{\min}$  85 ft/min

Bit 8 7/8" 14-14-14 Hole size 9 7/8"

Rate data

$Q_1$  300 GPM @  $P_{s1}$  2966 psi

$Q_2$  400 GPM @  $P_{s2}$  4883 psi

Find  $\alpha$

$$P_B = \frac{8.311E5\rho Q^2}{C_d^2 A_t^2}$$

$$P_{B1} = \frac{8.311E5 \cdot 15.5 \cdot 300^2}{.95^2 \cdot 45099^2} = 631.6 \text{ psi} \quad P_{B2} = \frac{8.311E5 \cdot 15.5 \cdot 400^2}{.95^2 \cdot 45099^2} = 1122.8 \text{ psi}$$

$$P_{f1} = 2966 - 631.6 = 2344.4 \text{ psi}$$

$$P_{f2} = 4883 - 1122.8 = 3760.2 \text{ psi}$$

$$\alpha = \frac{\log\left(\frac{P_{f2}}{P_{f1}}\right)}{\log\left(\frac{Q_2}{Q_1}\right)} = \frac{\log\left(\frac{3760.2}{2344.4}\right)}{\log\left(\frac{400}{300}\right)} = 1.66$$

Find  $Q_{\max}$  and  $Q_{\min}$

$$Q_{\max} = 1714 \cdot .8 \left(\frac{1600}{5440}\right) = 403 \text{ gpm} \quad \text{Based on pump}$$

$$Q_{\min} = 2.448(9.875^2 - 4.5^2) \frac{85}{60} = 268 \text{ gpm} \quad \text{Based on velocity}$$

Optimum friction pressure

$$P_{f_{opt}} = \left(\frac{1}{\alpha + 1}\right) P_{p_{\max}} = \left(\frac{1}{1.66 + 1}\right) 5440 = 2047 \text{ psi}$$

Optimum pressure drop at the bit

$$P_B = P_{p \max} - P_{f_{opt}} = 5440 - 2047 = 3393 \text{ psi}$$

Optimum flow rate

$$Q_{opt} = Q_a \text{anti log} \left[ \frac{1}{\alpha} \log \left( \frac{P_{f_{opt}}}{P_{f_{qa}}} \right) \right] = 300 \text{anti log} \left[ \frac{1}{1.66} \log \frac{2047}{2334} \right] = 227 \text{ gpm}$$

This is lower than the max and higher than min flow rates.

Optimum nozzle area

$$A_{nopt} = \sqrt{\frac{8.311E - 5 \cdot \rho \cdot Q_{opt}^2}{C_d^2 P_{B_{opt}}}} = \sqrt{\frac{8.311E - 5 \cdot 15.5 \cdot 227^2}{.95^2 3393}} = .15 \text{ in}^2$$

For 3 equal sized jets

$$d_{nopt} = 2 \sqrt{\frac{.15}{3\pi}} = .25 \text{ in}$$

The maximum jet impact force criterion assumes that the bottom-hole cleaning is achieved by maximizing the jet impact force with respect to the flow rate.

The impact force at the bottom of the hole can be derived from Newton's second law of motion

$$F_j = BQ\sqrt{P_B} \quad B = .01823C_d\sqrt{\rho} \quad Q \text{ in gpm } \rho \text{ in \#/gal}$$

$$P_B = P_s - P_f = P_s - CQ^\alpha$$

$$F_j = BQ\sqrt{P_s - CQ^\alpha}$$

limitations

- 1) maximum pump horsepower
- 2) maximum surface pressure

For the shallow portion of the well  $P_f$  is small and the flow rate requirement is large the impact force is limited only by the pump horsepower, therefore, the allowable surface pressure, expressed as

$$P_s = \frac{H_{p \max}}{Q}$$

substituting

$$F_j = BQ\sqrt{\frac{H_{p \max}}{Q} - CQ^\alpha} = B\sqrt{H_{p \max}Q - CQ^{\alpha+2}}$$

Differentiate and set to 0

$$\frac{dF_j}{dQ} = \frac{.5B[H_{p \max} - (\alpha + 2)CQ^{\alpha+1}]}{\sqrt{H_{p \max}Q - CQ^{\alpha+2}}} = 0$$

For a valid solution the numerator must be equal to zero.

Solve for the optimum friction pressure

$$P_{fopt} = \frac{1}{\alpha + 2} P_{sopt}$$

then solve for the optimum bit pressure

$$P_{Bopt} = P_{sopt} - P_{fopt} = \frac{\alpha + 1}{\alpha + 2} P_{sopt}$$

In the deeper sections of the well the friction pressure loss increases, while the flow rate requirement decreases. Therefore the impact force will be limited by the maximum allowed pump pressure,  $P_{smax}$ .

$$P_j = BQ\sqrt{P_{smax} - CQ^\alpha}$$

Differentiate and set to 0

$$\frac{dF_j}{dQ} = \frac{.5B[P_{smax} - (\alpha + 2)CQ^{\alpha+1}]}{\sqrt{P_{smax}Q - CQ^{\alpha+2}}} = 0$$

For a valid solution the numerator must be equal to zero.

$$P_{fopt} = \frac{2}{\alpha + 2} P_{smax}$$

Gives

$$P_{Bopt} = P_{smax} - P_{fopt} = \frac{\alpha}{\alpha + 2} P_{smax}$$

### Example

Same data as Hydraulic example

So  $\alpha=1.66$   $Q_{\max}=4.3$  gpm  $Q_{\min}=268$  gpm

At 12,000 feet the pump pressure is the limiting factor.

$$P_{fopt} = \frac{2}{\alpha + 2} P_{s\max} = \frac{2 \cdot 5440}{1.66 + 2} = 2975 \text{ psi}$$

$$P_{Bopt} = P_{s\max} - P_{fopt} = 5440 - 2975 = 2465 \text{ psi}$$

$$Q_{opt} = Q_a \text{anti log} \left[ \frac{1}{\alpha} \log \left( \frac{P_{fopt}}{P_{fqa}} \right) \right] = 300 \text{anti log} \left[ \frac{1}{1.66} \log \frac{2975}{2334} \right] = 347 \text{ gpm}$$

It is bounded by the min and max flow rates, so

$$A_{topt} = \sqrt{\frac{8.311E - 5 \cdot \rho \cdot Q_{opt}^2}{C_d^2 P_{Bopt}}} = \sqrt{\frac{8.311E - 5 \cdot 15.5 \cdot 347^2}{.95^2 \cdot 2465}} = .26 \text{ in}^2$$

$$d_{nopt} = 2 \sqrt{\frac{.26}{3\pi}} = .332 \text{ in} = 10.6 / 32''$$

3 - 11 jets have an area of .27in<sup>2</sup>.

Section 4.13 in text, pages 156, 157

## Cuttings Lifting

Rock weights about 21 ppg, so it will fall in any fluid that has a lower density. The rate that the cutting fall in the drilling fluid is the slip velocity. To maintain good hole cleaning the velocity of the drilling fluid has to be greater than the slip velocity of the cuttings. The slip velocity depends on the difference in densities, viscosity of the fluid and the size of the cuttings.

$$v_s = 113.4 \left[ \frac{d_p (\rho_p - \rho_f)}{C_D \rho_f} \right]^{0.5}$$

$d_p$  the diameter of the cuttings inches

$\rho_p$  the density of the cuttings 21 ppg

$C_D$  Drag coefficient

### Particle Reynolds number

$$R_p = \frac{15.47 \rho v_s d_p}{\mu}$$

which gives

$$C_D = \frac{40}{R_p}$$

Substituting in the first equation

$$v_s = \frac{4980 d_p^2 (\rho_p - \rho_f)}{\mu}$$

For values of  $R_p$  greater than 1 which means laminar flow around the particle the drag coefficient can be found using

$$C_D = \frac{22}{R_p^5}$$

So the slip velocity equation becomes

$$v_s = \frac{175 d_p (\rho_p - \rho_f)^{0.667}}{\rho_f^{0.333} \mu^{0.333}}$$

## Designing the hydraulic system

- 1) Break the well down into sections, hole size, drilling fluid changes and depth etc. Design the drilling fluids for each.
- 2) Calculate the maximum pump rate using the pump specifications.
- 3) Calculate the friction loss in the pipe and annulus for each section using 2 flow rates. From this calculate the flow exponent  $\alpha$  for that section.
- 4) Using this flow exponent optimize the bit hydraulics.
- 5) When drilling confirm your plan by finding the  $\alpha$  by measuring the friction pressure at 2 pump rates.
- 6) Find the annular velocity at the optimal rate and compare it to the slip velocity, verify that this rate will clean the hole.
- 7) Calculate the pressures and horsepower required to pump the optimal rate for the bit and verify the equipment can handle it.
- 8) Trial and error may be required to find the optimal rate and jet sizes.