

Non-Newtonian fluids

Two models for non-newtonian fluids, Bingham fluid ($\tau = \tau_y + \mu_p \gamma$) and Power Law fluid ($\tau = K\gamma^n$).

Laminar Flow
Pipe

$$\text{Bingham} \quad \Delta P_f = \frac{\mu_p v}{1500d^2} + \frac{\tau_y}{225d} \quad \text{oilfield units}$$

μ_p plastic viscosity cp
 v average velocity ft/sec
 d ID of the pipe in inches
 τ_y Yield value in lbs/100 ft²

$$\mu_p = \theta_{600} - \theta_{300}$$

$$\tau_y = \theta_{300} - \mu_p$$

$$\text{Power Law} \quad \Delta P_f = \frac{kv^n \left(\frac{3+1}{n} \right)}{144,000d^{1+n}}$$

K consistency index

n power law index

$$n = 3.32 \log \left(\frac{\theta_{600}}{\theta_{300}} \right)$$

$$K = \frac{510\theta_{300}}{511^n}$$

Example

Bingham

$$\mu_p = \theta_{600} - \theta_{300} = 55 - 35 = 20cp$$

$$\tau_y = \theta_{300} - \mu_p = 35 - 20 = 15\#/100ft^2$$

$$\Delta P_f = \frac{\mu_p v}{1500d^2} + \frac{\tau_y}{225d} = \frac{20 \cdot 4.4}{1500 \cdot 4^2} + \frac{15}{225 \cdot 4} = .02psi/ft$$

Power Law

$$n = 3.32 \log\left(\frac{\theta_{600}}{\theta_{300}}\right) = 3.32 \log\left(\frac{55}{35}\right) = .652$$

$$K = \frac{510\theta_{300}}{511^n} = \frac{510 \cdot 35}{511^{.652}} = 306cp_e$$

$$\Delta P_f = \frac{kv^n \left(\frac{3+1/n}{.0416}\right)}{144,000d^{1+n}} = \frac{306 \cdot 4.4^{.652} \left(\frac{3+1/.652}{.0416}\right)}{144000 \cdot 4^{1+.652}} = .06psi/ft$$

Annulus

$$\text{Bingham} \quad \Delta P_{fa} = \frac{\mu_p v_a}{1000(d_h - d_{od})^2} + \frac{\tau_y}{200(d_h - d_{od})}$$

μ_p plastic viscosity cp

v average velocity ft/sec

d ID of the pipe in inches

τ_y Yield value in lbs/100 ft²

$$\mu_p = \theta_{600} - \theta_{300}$$

$$\tau_y = \theta_{300} - \mu_p$$

$$\text{Power Law} \quad \Delta P_{fa} = \frac{kv_a^n}{144000(d_h - d_{od})^{n+1}} \left(\frac{2 + 1/n}{.0208} \right)^n$$

K consistency index

n power law index

$$n = 3.32 \log \left(\frac{\theta_{600}}{\theta_{300}} \right)$$

$$K = \frac{510\theta_{300}}{511^n}$$

Example

Same fluid as previous example, min velocity 100 ft/min

Drill Pipe 5" and 1000' of collars 7" TD 10,000' hole 8 7/8"

Bingham

$$v_{adp} = 100 / 60 = 1.67 \text{ ft / sec}$$

$$\mu_p = \theta_{600} - \theta_{300} = 55 - 35 = 20 \text{ cp}$$

$$\tau_y = \theta_{300} - \mu_p = 35 - 20 = 15 \# / 100 \text{ ft}^2$$

$$\Delta P_{fa} = \frac{\mu_p v_a}{1000(d_h - d_{od})^2} + \frac{\tau_y}{200(d_h - d_{od})} = \frac{20 \cdot 1.67}{1000(8.875 - 5)^2} + \frac{15}{200(8.875 - 5)}$$

$$\Delta P_{adp} = .022 \text{ psi / ft}$$

$$v_{adc} = 2.4 \text{ ft / sec}$$

$$\Delta P_{fa} = \frac{\mu_p v_a}{1000(d_h - d_{od})^2} + \frac{\tau_y}{200(d_h - d_{od})} = \frac{20 \cdot 2.4}{1000(8.875 - 7)^2} + \frac{15}{200(8.875 - 7)}$$

$$\Delta P_{adc} = .054 \text{ psi / ft}$$

$$P_f = P_p + P_{adp} + P_{adc} = \Delta P_p L + \Delta P_{adp} L + \Delta P_{adc} L$$

$$P_f = .02 \cdot 10000 + .022 \cdot 9000 + .054 \cdot 1000 = 452 \text{ psi}$$

Power Law

$$n = 3.32 \log \left(\frac{\theta_{600}}{\theta_{300}} \right) = 3.32 \log \left(\frac{55}{35} \right) = .652$$

$$K = \frac{510 \theta_{300}}{511^n} = \frac{510 \cdot 35}{511^{.652}} = 306 \text{ eqcp}$$

$$\Delta P_{fa dp} = \frac{kv_a^n}{144000(d_h - d_{od})^{n+1}} \left(\frac{2 + \frac{1}{n}}{.0208} \right)^n = \frac{306 \cdot 1.67^{.652}}{144000(8.875 - 5)^{1.652}} \left(\frac{2 + \frac{1}{.652}}{.0208} \right)^{.652}$$

$$\Delta P_{adp} = .009 \text{ psi} / \text{ft}$$

$$\Delta P_{fa dc} = \frac{kv_a^n}{144000(d_h - d_{od})^{n+1}} \left(\frac{2 + \frac{1}{n}}{.0208} \right)^n = \frac{306 \cdot 2.4^{.652}}{144000(8.875 - 7)^{1.652}} \left(\frac{2 + \frac{1}{.652}}{.0208} \right)^{.652}$$

$$\Delta P_{adp} = .01 \text{ psi} / \text{ft}$$

$$P_f = P_p + P_{adp} + P_{adc} = \Delta P_p L + \Delta P_{adp} L + \Delta P_{adc} L$$

$$P_f = .02 \cdot 10000 + .009 \cdot 9000 + .01 \cdot 1000 = 291 \text{ psi}$$

Turbulent Flow

Equivalent Newtonian Viscosity

Bingham

$$\mu_{eBA} = \mu_p + \frac{5(D_h - D_{od})\tau_y}{v_p} \qquad \mu_{eBp} = \mu_p + \frac{20D_{id}\tau_y}{3v_p}$$

Power Law

$$\mu_{ePp} = \frac{kv_p^{n-1}}{96D_{id}^{n-1}} \left[\frac{3 + (1/n)}{.0416} \right]^n \qquad \mu_{ePp} = \frac{kv_p^{n-1}}{144(D_h - D_{od})} \left[\frac{2 + (1/n)}{.0208} \right]^n$$

Apparent Viscosity Bingham

$$\mu_a = \mu_p + \frac{6.66\tau_y}{v} \qquad \mu_a = \mu_p + \frac{5\tau_y(d-d)}{v}$$

Power Law

$$\mu_a = \frac{Kd^{(1-n)}}{96v^{1-n}} \left(\frac{3 + 1/n}{.0416} \right)^n$$

These viscosities can be used in place of the newtonain viscosity in N_{Re} .

Pressure loss equations

Pipe

$$\text{Bingham} \qquad \Delta P_f = \frac{f\rho v^2}{25.8d} \qquad \Delta P_f = \frac{\rho^{.75} v^{1.75} \mu_p^{.25}}{1800d^{1.25}}$$

$$\text{Using flow rate} \qquad \Delta P_f = \frac{.000077\rho^8 \mu_p^2 Q^{1.7}}{d^{4.8}}$$

Power Law $\Delta P_f = \frac{f\rho v^2}{25.8d}$

Annulus

Bingham

$$\Delta P_f = \frac{f\rho v^2}{21.1(d_2 - d_1)}$$

$$\Delta P_f = \frac{\rho^{.75} v^{1.75} \mu_p^{.25}}{1396(d_2 - d_1)^{1.25}}$$

Power Law

$$\Delta P_f = \frac{f\rho v^2}{21.1(d_2 - d_1)}$$

Example

Mud $\theta_{600}=55$ $\theta_{300}=35$ 9.5 #/gal
Drill String DP 5"od 4"id collars 7"od 4"id
Hole 8 7/8" TD 10,000'
Min velocity 100 ft/min

Find the flow rate and velocities

$$Q = v_{a \min} A_{a \max} = v \frac{\pi}{4} (d_{hd}^2 - d_{pod}^2)$$

$$Q = 100 \frac{\pi}{4} (8.875_{hd}^2 - 5_{pod}^2) 0.52 \approx 219 \text{ gpm}$$

$$Q = 219 \text{ gpm} \cdot .13368 \text{ ft}^3 / \text{gal} = 44.11 / 60 = .488 \text{ ft}^3 / \text{sec}$$

$$v_p = \frac{Q}{A_{id}} = \frac{.488 \text{ ft}^3 / \text{sec}}{8.73E - 2 \text{ ft}^2} = 5.6 \text{ ft} / \text{sec}$$

$$v_{ac} = \frac{Q}{A_{ac}} = \frac{.488 \text{ ft}^3 / \text{sec}}{.162 \text{ ft}^2} = 3 \text{ ft} / \text{sec}$$

$$v_{adp} = \frac{100 \text{ ft} / \text{sec}}{60} = 1.67 \text{ ft} / \text{sec}$$

$$n = 3.32 \log \left(\frac{\theta_{600}}{\theta_{300}} \right) = 3.32 \log \left(\frac{55}{35} \right) = .652$$

$$K = \frac{510 \theta_{300}}{511^n} = \frac{510 \cdot 35}{511^{.652}} = 306 \text{ eqcp}$$

$$\mu_p = \theta_{600} - \theta_{300} = 55 - 35 = 20 \text{ cp}$$

$$\tau_y = \theta_{300} - \mu_p = 35 - 20 = 15 \# / 100 \text{ ft}^2$$

Check flow regimes

$$\text{Pipe } N_{Re} = \frac{928\rho v d}{\mu_p} = \frac{928 \cdot 9.5 \cdot 4}{20} = 14900 \text{ turbulent}$$

$$\text{Annulus Collars } N_{Re} = \frac{757\rho v(d_2 - d_1)}{\mu_p} = \frac{757 \cdot 9.5 \cdot 3(8.875 - 7)}{20} = 2022$$

$$\text{Annulus DP } N_{Re} = \frac{757\rho v(d_2 - d_1)}{\mu_p} = \frac{757 \cdot 9.5 \cdot 1.67(8.875 - 5)}{20} = 2326$$

Use laminar for the annulus

Pipe calculations

$$\Delta P_f = \frac{\rho^{.75} v^{1.75} \mu_p^{.25}}{1800 d^{1.25}} = \frac{9.5^{.75} \cdot 5.6^{1.75} \cdot 20^{.25}}{1800 \cdot 4^{1.25}} = .02 \text{ psi / ft}$$

$$P_{fp} = \Delta P_{fp} L = .02 \cdot 10000 = 200 \text{ psi}$$

Annulus calculations

$$\Delta P_{fadc} = \frac{\mu_p v_a}{1000(d_h - d_{od})^2} + \frac{\tau_y}{200(d_h - d_{od})} = \frac{20 \cdot 2.4}{1000(8.875 - 7)^2} + \frac{15}{200(8.875 - 7)}$$

$$\Delta P_{adc} = .054 \text{ psi / ft}$$

$$\Delta P_{fada} = \frac{\mu_p v_a}{1000(d_h - d_{od})^2} + \frac{\tau_y}{200(d_h - d_{od})} = \frac{20 \cdot 1.67}{1000(8.875 - 5)^2} + \frac{15}{200(8.875 - 5)}$$

$$\Delta P_{adp} = .022 \text{ psi / ft}$$

$$P_{fa} = \Delta P_{fadc} L + \Delta P_{fada} L = .054 \cdot 1000 + .022 \cdot 9000 = 252 \text{ psi}$$

$$P_{f_{sur}} = L_{eq} \Delta P_{fp} = 816 \cdot .02 = 16 \text{ psi} \quad \text{page 160}$$

$$P_{f_{tot}} = P_{f_{sur}} + P_{fp} + P_{fa} = 16 + 252 + 200 = 468 \text{ psi}$$

$$HHP = \frac{PQ}{1714} = \frac{468 \cdot 219}{1714} = 60 \text{ hp}$$

Cuttings Lifting

Rock weights about 21 ppg, so it will fall in any fluid that has a lower density. The rate that the cutting fall in the drilling fluid is the slip velocity. To maintain good hole cleaning the velocity of the drilling fluid has to be greater than the slip velocity of the cuttings. The slip velocity depends on the difference in densities, viscosity of the fluid and the size of the cuttings.

$$v_s = 113.4 \left[\frac{d_p (\rho_p - \rho_f)}{C_D \rho_f} \right]^{.5}$$

d_p the diameter of the cuttings inches

ρ_p the density of the cuttings 21 ppg

C_D Drag coefficient

Particle Reynolds number

$$R_p = \frac{15.47 \rho v_s d_p}{\mu}$$

which gives

$$C_D = \frac{40}{R_p}$$

Substituting in the first equation

$$v_s = \frac{4980 d_p^2 (\rho_p - \rho_f)}{\mu}$$

For values of R_p greater than 1 which means laminar flow around the particle the drag coefficient can be found using

$$C_D = \frac{22}{R_p^5}$$

So the slip velocity equation becomes

$$v_s = \frac{175 d_p (\rho_p - \rho_f)^{.667}}{\rho_f^{.333} \mu^{.333}}$$

Designing the hydraulic system

- 1) Break the well down into sections, hole size, drilling fluid changes and depth etc. Design the drilling fluids for each.
- 2) Calculate the maximum pump rate using the pump specifications.
- 3) Calculate the friction loss in the pipe and annulus for each section using 2 flow rates. From this calculate the flow exponent α for that section.
- 4) Using this flow exponent optimize the bit hydraulics.
- 5) When drilling confirm your plan by finding the α by measuring the friction pressure at 2 pump rates.
- 6) Find the annular velocity at the optimal rate and compare it to the slip velocity, verify that this rate will clean the hole.
- 7) Calculate the pressures and horsepower required to pump the optimal rate for the bit and verify the equipment can handle it.
- 8) Trial and error may be required to find the optimal rate and jet sizes.

Homework

Mud $\theta_{600}=62$ $\theta_{300}=32$ 9.9 #/gal

Drill String DP 4 1/2"od 3.83" id collars 6"od 3.5" id

Hole 8" Depth 5,000'

Min velocity 125 ft/min

Calculate the friction drop of the system.

