2 Derivative Algorithms

Three algorithms are provided to compute the derivative \( \frac{d\Delta p}{d(\ln\Delta t)} \). These are (with increasing complexity): (i) two-points, (ii) three-consecutive-points, and (iii) three-smoothed-points method.

2.1 Two-Points Method

This method computes the logarithmic derivative at point-\( j \) from the two neighboring points, the point before \((j-1)\) and the point after \((j+1)\), see Fig. 2.1. The formula is

\[
\left( \frac{d\Delta p}{d(\ln\Delta t)} \right)_j = \frac{\Delta p_{j+1} - \Delta p_{j-1}}{\ln(\Delta t_{j+1}) - \ln(\Delta t_{j-1})} = \frac{\Delta p_{j+1} - \Delta p_{j-1}}{\ln(\Delta t_{j+1}/\Delta t_{j-1})}, \quad j \geq 2,
\]

where
- \( \Delta p \) = pressure change; \( p_i - p_{wfs}(\Delta t) \) for drawdown; \( p_{ws}(\Delta t) - p_{wfs} \) for buildup,
- \( p_i \) = initial reservoir pressure,
- \( p_{wfs} \) = well flowing pressure for drawdown; \( p_{ws} \) = well flowing pressure at the time of shut-in for buildup,
- \( \Delta t \) = elapsed time; \( \Delta t_{sh} \) = shut-in time, or Horner time, or Agarwal equivalent time, or superposition time for buildup,
- \( j \) = data point index.

Note that \( \ln(\Delta t_{j+1}/\Delta t_{j-1}) \) is always positive since \( \Delta t_{j+1} \) is always greater than \( \Delta t_{j-1} \). The computed derivative will never be negative unless \( \Delta p_{j+1} \) is smaller than \( \Delta p_{j-1} \).

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**Fig. 2.1 - Derivative Algorithms - 2-Points and 3-Consecutive-Points.**
2.2 Three-Consecutive-Points Method

This method uses one point before and one point after the point of interest, point-$j$, calculates the corresponding derivatives, and places their weighted mean at the point considered [Bourdet et al., 1989], see Fig. 2.1. The formula is

$$
\left( \frac{d\Delta p}{d \ln \Delta t} \right)_j = \frac{\Delta p_j - \Delta p_L}{\ln(\Delta t_i) - \ln(\Delta t_L)} \frac{\ln(\Delta t_R) - \ln(\Delta t_j)}{\ln(\Delta t_R) - \ln(\Delta t_L)} + \frac{\Delta p_R - \Delta p_j}{\ln(\Delta t_R) - \ln(\Delta t_i)} \frac{\ln(\Delta t_R) - \ln(\Delta t_j)}{\ln(\Delta t_R) - \ln(\Delta t_L)}, \quad j \geq 2,
$$

$$
= \frac{\Delta p_j - \Delta p_L}{\ln(\Delta t_i / \Delta t_L)} \frac{\ln(\Delta t_R / \Delta t_j)}{\ln(\Delta t_R / \Delta t_L)} + \frac{\Delta p_R - \Delta p_j}{\ln(\Delta t_R / \Delta t_j)} \frac{\ln(\Delta t_R / \Delta t_L)}{\ln(\Delta t_R / \Delta t_L)}, \quad j \geq 2,
$$

where

$L = j-1$, left to point-$j$,

$R = j+1$, right to point-$j$.

The above definitions of $L$ and $R$ means that three consecutive points are used in this method.

2.3 Three-Smoothed-Points Method

This method basically is the same as the previous three-consecutive-points method (i.e., same formula) but different in that the point-$L$ and point-$R$ chosen may not be consecutive. Determination of point-$L$ and point-$R$, depends on the specification of a "window width $W$" ($W \geq 0$) with $W$ refers to a length of $\ln \Delta t$ or natural logarithmic of other time functions. The algorithm chooses point-$L$ and point-$R$ as being the first ones such that

$$
[\ln(\Delta t_j) - \ln(\Delta t_L)] = \ln(\Delta t_j / \Delta t_L) > W, \text{ and}
$$

$$
\ln(\Delta t_R) - \ln(\Delta t_j) = \ln(\Delta t_R / \Delta t_j) > W,
$$

(i.e., just beyond the window width), see Fig. 2.2. Common values of $W$ are 0 up to 0.5 in extreme cases [Bourdet et al., 1989]. Note that $W=0$ is equivalent to the three-consecutive-points described previously. The primary intention of this method is to reduce noise. A compromise, however, must be made between the smoothness of the derivative and the possible distortion resulted from over-smoothing.

2.4 Relation Between Two-Points and Three-Points Methods

The three-points methods (both consecutive and smoothed) reduce to the two-points method if

$$
\frac{\ln(\Delta t_R / \Delta t_I)}{\ln(\Delta t_I / \Delta t_L)} = \frac{\ln(\Delta t_j / \Delta t_L)}{\ln(\Delta t_R / \Delta t_j)},
$$

which is equivalent to

$$
\Delta t_j / \Delta t_L = \Delta t_R / \Delta t_I, \text{ or } \Delta t_j = \sqrt{\Delta t_I (\Delta t_R)}.
$$

Thus the three-points method is equivalent to the two-points method if the ratios of the succeeding chosen $\Delta t$'s are the same.
2.5 Example

Consider the $\Delta p$ and $\Delta t$ data given in the first three columns of the following table. The pressure derivatives at data point 3 computed by the discussed three methods are also given in the same table. Detailed procedures are given following the table.

<table>
<thead>
<tr>
<th>Data Point $j$</th>
<th>Elapsed Time $\Delta t$ (hours)</th>
<th>Pressure Change $\Delta p$ (psi)</th>
<th>$\ln(\Delta t_j/\Delta t_R)$ (Absolute Value)</th>
<th>$d\Delta p/d(\ln\Delta t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Two-Points Method</td>
</tr>
<tr>
<td>1</td>
<td>0.09583</td>
<td>71.95</td>
<td>0.23181</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.10833</td>
<td>80.68</td>
<td>0.10920</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.12083</td>
<td>88.39</td>
<td>0</td>
<td>79.17</td>
</tr>
<tr>
<td>4</td>
<td>0.13333</td>
<td>97.12</td>
<td>0.09844</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.14583</td>
<td>104.24</td>
<td>0.18806</td>
<td></td>
</tr>
</tbody>
</table>