Modified Hagedron and Brown Method (mH-B)

This is an empirical two-phase flow correlation, the core of which is correlation for liquid hold-up. Griffith correlation is used for flow in the bubble flow region.

The mechanical energy balance equation used for the mH-B correlation

$$144\frac{dp}{dh} = \bar{\rho} + \frac{fw^2}{\left(7.413E10D^5\right)\bar{\rho}} + \bar{\rho}\frac{\Delta\left(u_m^2/2g_c\right)}{\Delta h} \qquad (2\text{-}5)$$

in oilfield units.-

Bubble flow regime exists if $\lambda_g < L_B$ where

$$L_B = 1.071 - .2218\left(\frac{u_m^2}{D}\right) \qquad (2\text{-}6)$$

if L_B <.13 set L_B to .13.

The input fraction of gas λ_g is $\lambda_g = u_{sg}/u_m$

To find the liquid hold-up that is needed to calculate the pressure gradient from correlations require the following dimensionless numbers

Liquid velocity $$N_{vl} = 1.938u_{sl}\sqrt[4]{\frac{\rho_l}{\sigma}} \qquad (2\text{-}7)$$

Gas velocity $$N_{vg} = 1.938u_{sg}\sqrt[4]{\frac{\rho_l}{\sigma}} \qquad (2\text{-}8)$$

Pipe diameter $$N_D = 120.872D\sqrt{\frac{\rho_l}{\sigma}} \qquad (2\text{-}9)$$

Liquid viscosity $N_L = .15726\mu_l \sqrt[4]{\frac{1}{\rho_l \sigma^3}}$ (2-10)

Where superficial velocities u_s in ft/sec, density ρ in lb_m/ft^3, surface tension σ in dynes/cm, diameter D in feet.

Using mH-B method

First find the flow regime.

Calculate mass flow rate w and density ρ as before.

Calculate the velocities u_l & u_g

$$u_l = \frac{q_l 5.614}{A_t 86400} \qquad u_g = \frac{q_l GLR \beta_g}{A_t 86400} \qquad u_m = u_l + u_g$$

Calculate the dimensionless numbers.

Using N_L find CN_L from fig. 2-3

Then find the group $\frac{N_{vl} p^{0.1}(CN_L)}{N_{vg}^{0.575} p_a^{0.1} N_D}$ find y_l/ψ from fig 2-4

p here is the absolute pressure at the location of interest.

Compute $\frac{N_{vg} N_L^{0.38}}{N_D^{2.14}}$ from fig 2-5 we get ψ

Get the liquid hold-up y_l $y_l = (y_l / \psi)/\psi$

Note: For low viscosity fluid ψ will generally be 1.

For f a 2 phase Reynolds number is be used

$$N_{Re} = \frac{2.2E-2w}{D\mu_l^{y_l}\mu_g^{(1-y_l)}}$$

f from Moody diagram.

Get pressure gradient from, assuming no kinetic energy

$$144\frac{dp}{dh} = \bar{\rho} + \frac{fw^2}{(7.413E10D^5)\bar{\rho}}$$

The Griffith correlation

For bubble flow Griffith uses a different hold-up correlation, bases the frictional pressure gradient on the in-situ average liquid velocity.

$$144\frac{dp}{dh} = \bar{\rho} + \frac{fw_l^2}{(7.413E10)D^5\rho_l y_l^2} \qquad \text{(2-11)}$$

The liquid hold-up is

$$y_l = 1 - \frac{1}{2}\left[1 + \frac{u_m}{u_s} - \sqrt{\left(1 + \frac{u_m}{u_s}\right)^2 - 4\frac{u_{sg}}{u_s}}\right] \qquad \text{(2-12)}$$

where slip velocity u_s is .8 ft/sec.

Reynolds number is based on in-situ liquid velocity

$$N_{Re} = \frac{D\bar{u}_l\rho_l}{\mu_l} \quad \text{or} \quad N_{Re} = \frac{2.2E-2w_l}{D\mu_l} \qquad \text{(2-13)}$$

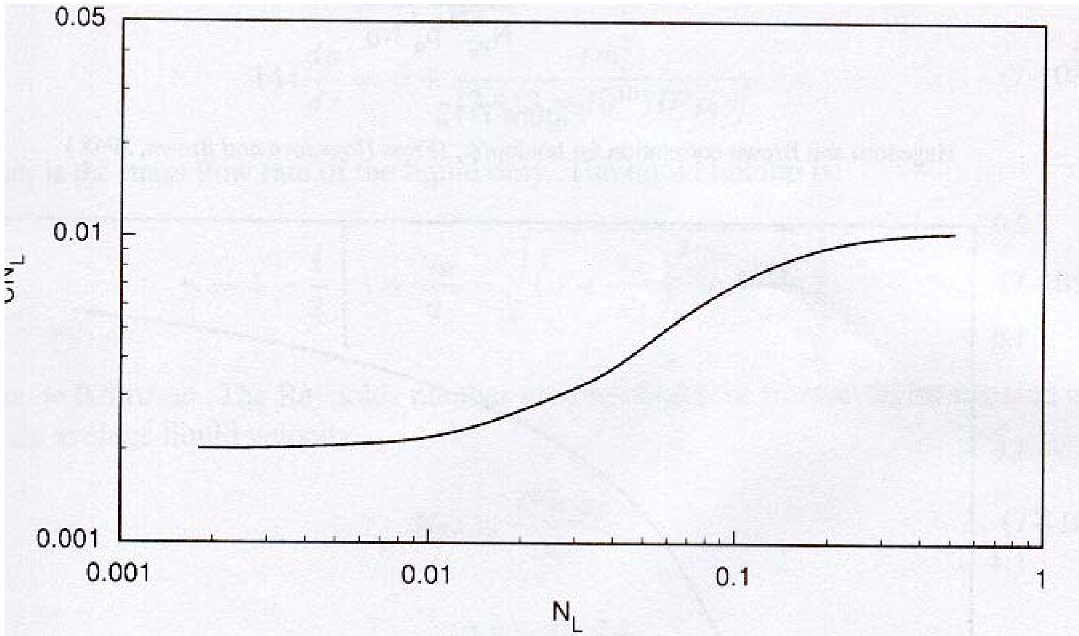


figure 2-3

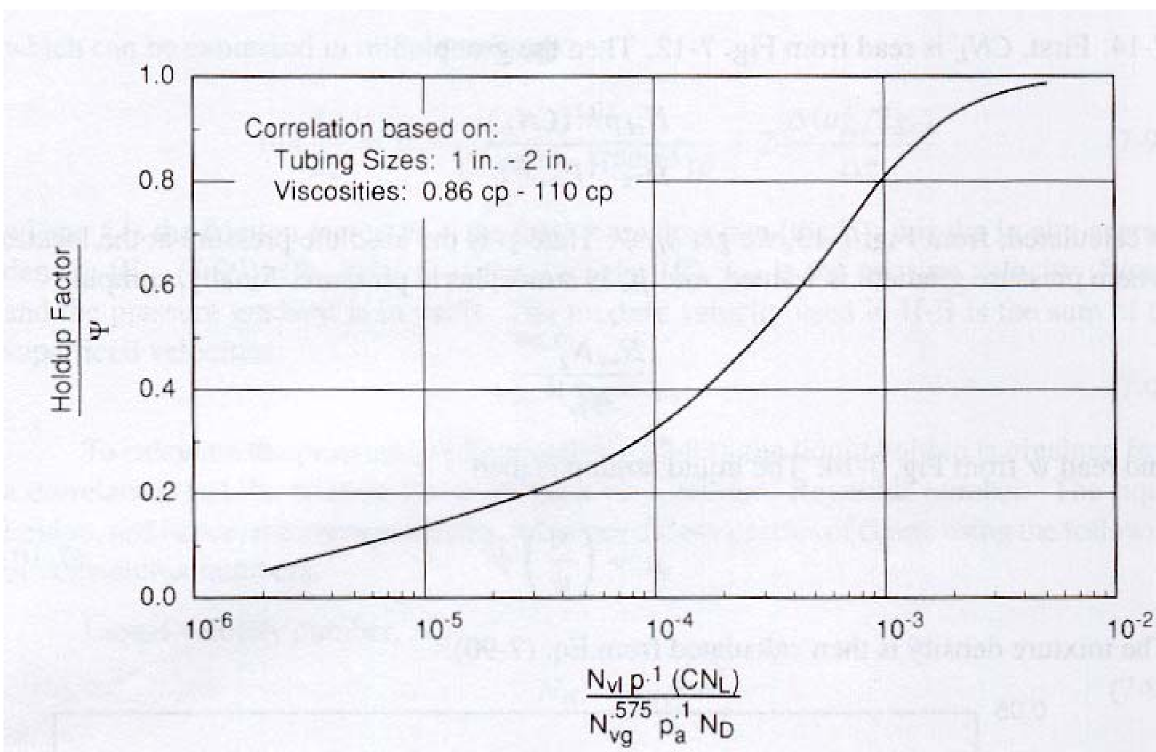


figure 2-4

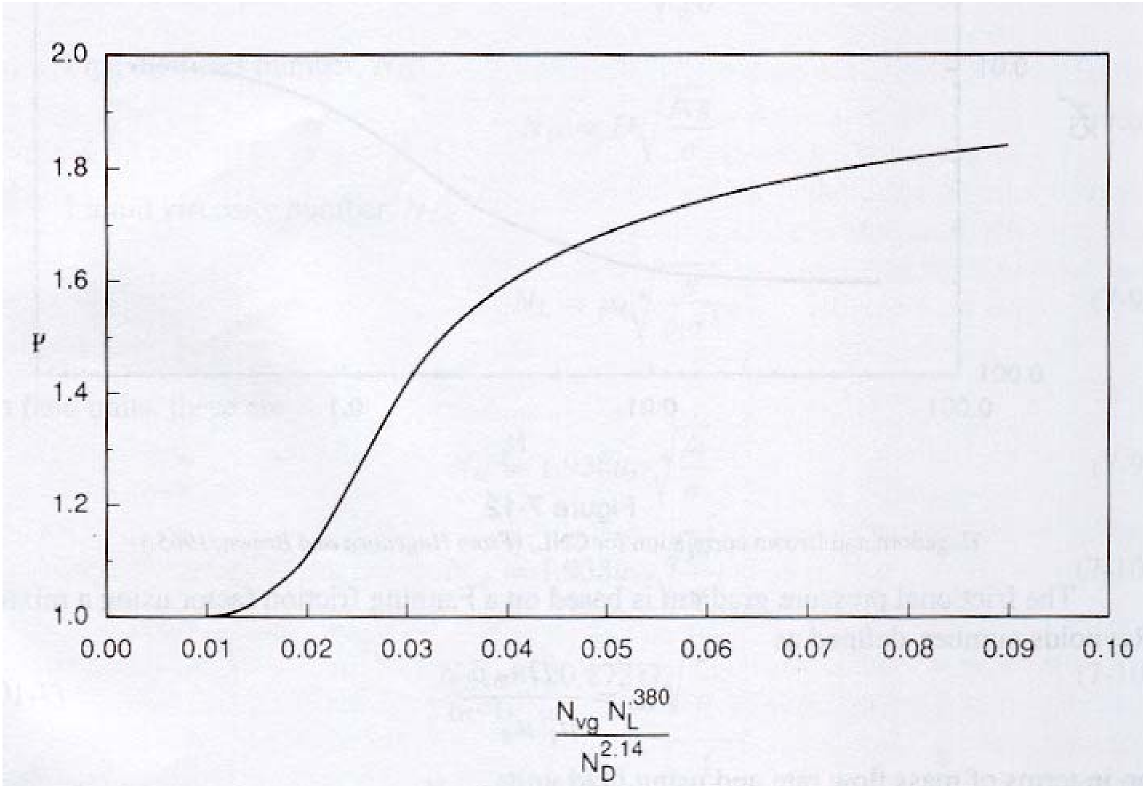


figure 2-5

Equations to replace figure 2-3

$$(CN_L) = 10^Y$$

$$Y = -2.69851 + 0.15841X_1 - 0.55100X_1^2 + 0.54785X_1^3 - 0.12195X_1^4$$

$$X_1 = \log[(N_L) + 3]$$

to replace figure 2-4

$$(y_L / \psi) = -0.10307 + 0.61777[\log(X_2) + 6] - 0.63295[\log(X_2) + 6]^2 +$$

$$+ 0.29598\ [\log(X_2) + 6]^3 - 0.0401\ [\log(X_2) + 6]^4$$

$$X_2 = \frac{N_{vL} p^{0.1} (CN_L)}{N_{vG}^{0.575} p_a^{0.1} N_D}$$

for figure 2-5

$$\psi = 0.91163 - 4.82176X_3 + 1232.25X_3^2 - 22253.6X_3^3 + 116174.3X_3^4$$

$$X_3 = \frac{N_{vG} N_L^{0.38}}{N_D^{2.14}}$$

if $X_3 > .01$

for $X_3 < .01$ then $\psi = 1$

and for *f*

$$\frac{1}{\sqrt{f_f}} = -4\log\left\{\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{\mathrm{Re}}}\log\left[\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{\mathrm{Re}}}\right)^{0.8981}\right]\right\}$$

Find the pressure gradient at 500 psi

Tubing 2"	q_o. = 400 bpd
22^o API oil	q_w.= 600 bpd
γ_w = 1.07	GLR = 500 cf/bbl
γ_g = .65	σ_o = 30 dynes/cm
σ_w = 70 dynes/cm	μ_o = 3 cp
μ_g = .015 cp	T = 105F

γ_o = 141.5/131.5 + 22 = .922

M = .4*.922*350 + .6*1.07*350 + 500*.65*.0764 = 378.6 lb/bbl

w = M*q = 378.6*1000 = 378610 lb/day

Find the flow regime

$$u_l = \frac{q_l 5.614}{A_t 86400} \qquad u_g = \frac{q_l GLR \beta_g}{A_t 86400} \qquad u_m = u_l + u_g$$

$$u_l = \frac{1000 \cdot 5.614}{.022 \cdot 86400} = 2.98\, ft/\sec \qquad u_g = \frac{1000 \cdot 500 \cdot .0298}{.022 \cdot 86400} = 7.84\, ft/\sec$$

$$u_m = u_l + u_g = 2.98 + 7.84 = 10.82\, ft/\sec$$

$$L_B = 1.071 - .2218\left(\frac{u_m^2}{D}\right) = 1.071 - .2218\left(\frac{10.82^2}{.166}\right) = -155.3$$

L_B<.13 so L_B =.13 $\qquad \lambda_g = u_{sg}/u_m = 7.84/10.82 = .725$

λ_g is larger than L_B so the flow regime is not bubble.

ρ_l = M_l/5.614[oil cut x β_o + water cut]
353.8/5.614[.4*1.043 + .6] = 61 lb/cf

ρ_g = M_g/ β_g{GLR – Oil cut x R_s}
24.8/ .0298{500 - .4*59}=1.75 lb/cf

$$N_{vl} = 1.938 u_{sl} \sqrt[4]{\frac{\rho_l}{\sigma}} = 1.938 \cdot 2.98 \sqrt[4]{\frac{61}{54}} = 6.89$$

$$N_{vg} = 1.938 u_{sg} \sqrt[4]{\frac{\rho_l}{\sigma}} = 1.938 \cdot 7.84 \sqrt[4]{\frac{61}{54}} = 15.66$$

$$N_D = 120.872 D \sqrt{\frac{\rho_l}{\sigma}} = 120.872 \cdot .166 \sqrt{\frac{61}{54}} = 21.4$$

$$N_L = .15726 \mu_l \sqrt[4]{\frac{1}{\rho_l \sigma^3}} = .15726 \cdot 2.98 \sqrt[4]{\frac{1}{61 \cdot 54^3}} = .0084$$

From chart 2-3 $CN_L = .0022$

$$\frac{N_{vl} p^{0.1} (CN_L)}{N_{vg}^{0.575} p_a^{0.1} N_D} = \frac{6.89 \cdot 500^{.1} .0022}{15.66^{.575} 14.7^{.1} 21.4} = 2.1E - 4$$

From chart 2-4 $y_l / \psi = .47$

$$\frac{N_{vg} N_L^{0.38}}{N_D^{2.14}} = \frac{15.66 \cdot .0084^{.38}}{21.4^{2.14}} = .0036$$

From chart 2-5 $\Psi = 1$ so $y_l = .47$

$$N_{\mathrm{Re}} = \frac{2.2E - 2w}{D \mu_l^{y_l} \mu_g^{(1-y_l)}} = \frac{2.2E - 2 \cdot 378610}{.166 \cdot 3^{.47} \cdot .015^{(1-.47)}} = 2777288$$

From the graph $f = .005$

$$144 \frac{dp}{dh} = \bar{\rho} + \frac{fw^2}{(7.413E10 D^5)\bar{\rho}} = 19 + \frac{.005 \cdot 378610^2}{7.412E10 \cdot .166^5 19} = 3.63/144 = .028 \, psi/ft$$

Homework

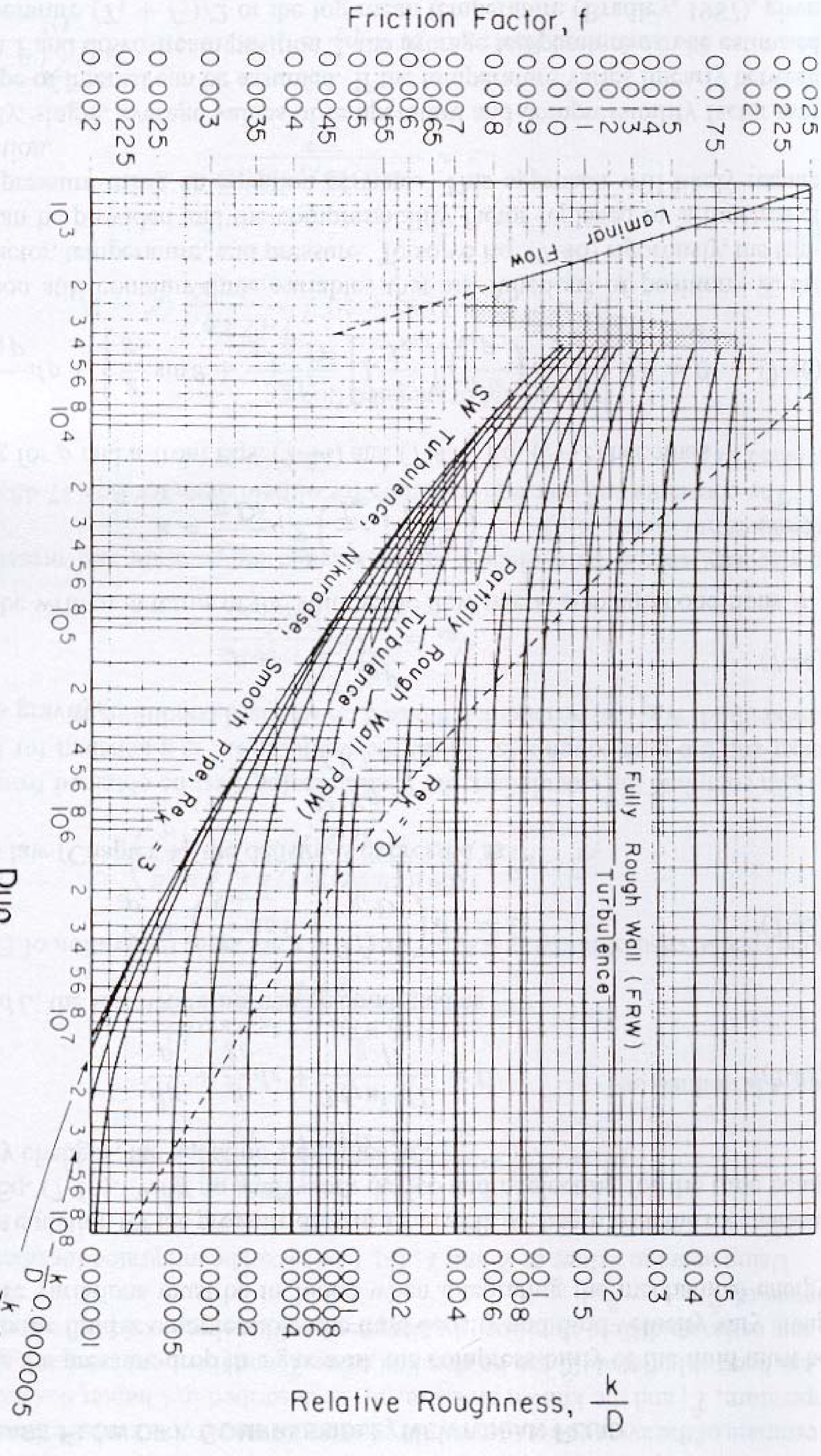
Use the data from the previous homework and solve the pressure gradient at 1000 psi.

Additional data

$\mu_o = 4$ cp $\mu_g = .02$ cp

$\sigma_o = 30$ dynes/cm $\sigma_w = 70$ dynes/cm

T = 145F



Friction Factor, f
0.002
0.00225
0.0025
0.003
0.0035
0.004
0.0045
0.005
0.0055
0.006
0.0065
0.007
0.008
0.009
0.010
0.011
0.012
0.013
0.014
0.015
0.0175
0.020
0.0225
0.025
Laminar Flow
SW Turbulence, Nikuradse
Smooth Pipe, Re* = 3
Partially Rough Turbulence Wall (PRW)
Re* = 70
Fully Rough Wall Turbulence (FRW)
Reynolds Number, NRe = Dup/μ
Relative Roughness, k/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
k/D 0.000005
k/D 0.000001