Flow in Gas Wells

Assumptions

- Kinetic Energy change is negligible
- Flow is steady state and isothermal
- No work done by the gas flow

R. V. Smith derived the equation for vertical flow

$$Q = 200{,}000\left[\frac{d^5}{\gamma_g T_a z_a fX}\left(P_2^2 - e^S P_1^2\right)\frac{S}{e^S - 1}\right]^{.5} \quad (2\text{-}18)$$

Q = Flow rate, mscfpd
Ta = Average Temperature, oR
d = ID of pipe, inches
P1 = Sand face pressure, pwf , at depth X, psi
P2 = Well head pressure, THP, psi
S = .0375γX/Taza
γ_g = gas gravity
f = friction factor

This equation also assumes a constant z and temperature.
The gravity of the fluid when a liquid is present is found be the following equation

$$\gamma = \gamma_g + \frac{4591\gamma_o / GLR}{1 + 1123 / GLR} \quad (2\text{-}19)$$

There is two ways of finding the friction factor required for equation 2-18. The first is using a table using the ID of the tubing and γQ/μ to get the value. Cullender & Binckley showed that the absolute roughness of .0006 and .00065 will work for all pipe from 1.25 to 8.625”. So they expressed the friction factor in terms of Q, γ, d, & μ,

$$f = 30.9208 \times 10^{-3}\,\frac{Q^{-.065} d^{-.056} \gamma^{-.065}}{\mu^{-.065}} \quad (2\text{-}20)$$

Moody friction factors

Rewrite equation 2-18 for Moody Friction Factors

$$P_{wf}^2 - e^S P_{TH}^2 = \frac{6.67 \times 10^{-4} \left(Q T_a z_a\right)^2 \left(e^S - 1\right) f}{d^5} \tag{2-21}$$

$$N_{\text{Re}} = 20.09 \frac{\gamma_g Q}{d\mu} \tag{2-22}$$

Cullender & Binkley Friction Group

Rewrite equation 2-18 for Fanning Friction Factor

$$P_{wf}^2 = e^S P_{TH}^2 + R^2 \tag{2-23}$$

define R $$R^2 = \left(F_{dnew} Q\right)^{1.935} \left(e^s - 1\right) \tag{2-24}$$

and $$F_D = 65.710 \big/ d^{2.614} \tag{2-25}$$

$$F_{Dnew} = F_D \left(\frac{Tz}{540}\right)^{1.0336} \left(\frac{.72\mu}{.0109\gamma}\right)^{.0336} \tag{2-26}$$

Q is mmscfpd, P^2 is in 1/1000

Poettmann derived an equation to take in to account the changing z factor.

$$X = \frac{X_s}{1 + .9521 \times 10^{-6} \times fQ^2 G^2 X_s^2 / D^5 (\Delta P)^2} \qquad (2\text{-}27)$$

$$X_s = \frac{53.241 T_a}{G} \left(\int_{.2}^{P_{r,2}} \frac{z}{P_r} dP_r - \int_{.2}^{P_{r,1}} \frac{z}{P_r} dP_r \right) \qquad (2\text{-}28)$$

tables are used to solve the integrals in these equations. This method requires for the pressure to computed at several depths and then plotted pressure vs. depth. The value for the BHP is found from this graph.

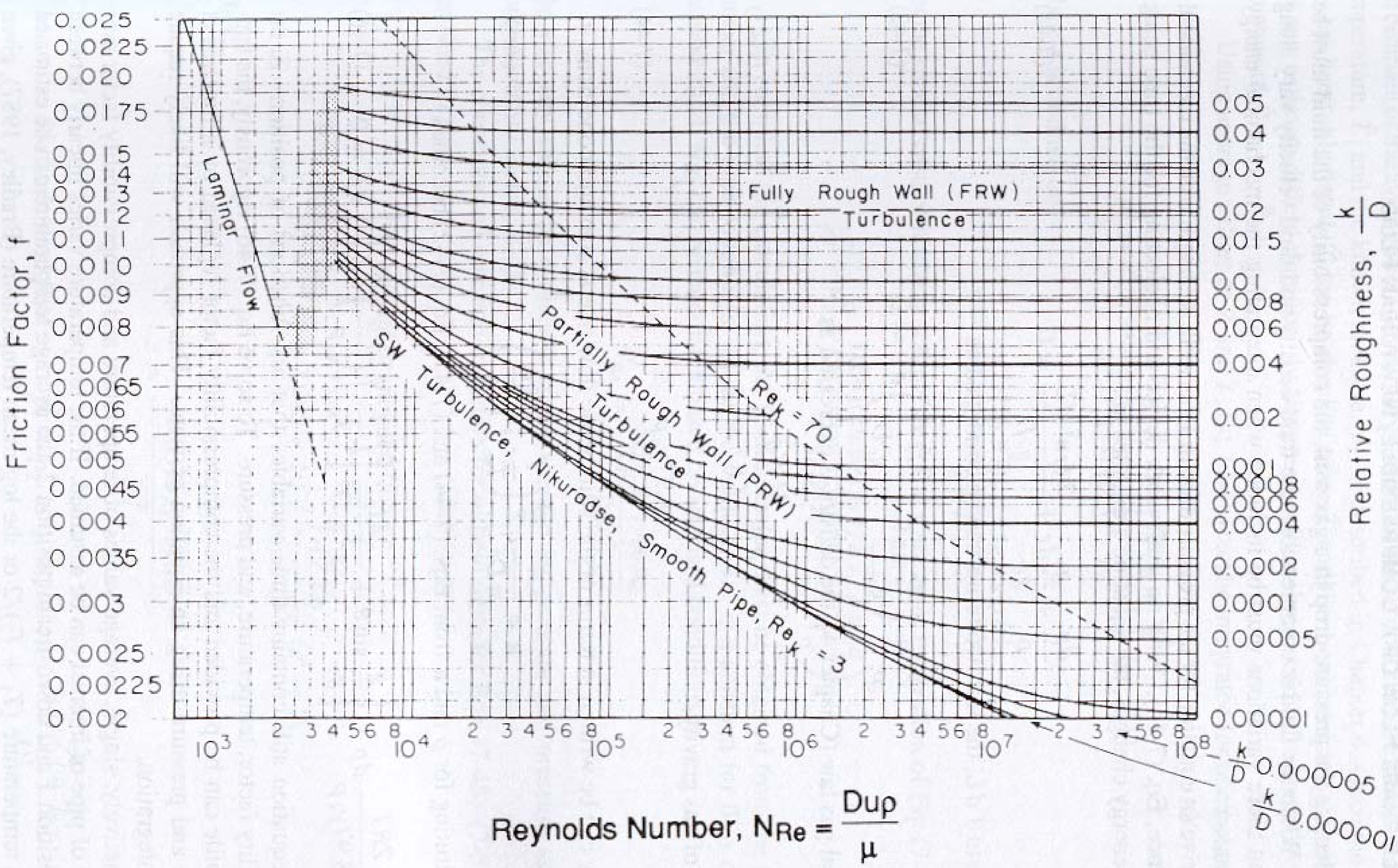
To calculated the static bottom hole pressure of a gas well use the following:

$$P_s - P_{WH} = P_{WH} \left(\exp \frac{.01877 GX}{T_a z_a} - 1 \right) \qquad (2\text{-}29)$$

or

$$\int_{.2}^{P_{r,2}} \frac{z}{P_r} dP_r = 0.01877 \frac{XG}{T_a} + \int_{.2}^{P_{r,1}} \frac{z}{P_r} dP_r \qquad (2\text{-}30)$$

X is the Δh, P_1 is the wellhead pressure, P_2 is the bottom hole pressure, psia.


Friction Factor, f
0.025
0.0225
0.020
0.0175
0.015
0.014
0.013
0.012
0.011
0.010
0.009
0.008
0.007
0.0065
0.006
0.0055
0.005
0.0045
0.004
0.0035
0.003
0.0025
0.00225
0.002
Laminar Flow
Fully Rough Wall (FRW) Turbulence
Partially Rough Wall (PRW) Turbulence
Rek = 70
SW Turbulence, Nikuradse, Smooth Pipe, Rek = 3
Relative Roughness, k/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
k/D 0.000005
k/D 0.000001
10^3 2 3 4 5 6 8 10^4 2 3 4 5 6 8 10^5 2 3 4 5 6 8 10^6 2 3 4 5 6 8 10^7 2 3 4 5 6 8 10^8
Reynolds Number, NRe = Duρ/μ

Wellbore Flow
Summary

Flow regimes

Flow in the wellbore is not single phase in most cases but a multiphase flow of gas and liquid. This breaks down to 5 flow regimes with varying ratios of liquid to gas. Each of these regimes has different effects on the friction generated by the flow up the well.

Methods

Poettman & Carpenter Method

Assumes negligible kinetic energy and external work done by the flow. The energy loss is caused by slippage and frictional effects.

$$144\frac{\Delta p}{\Delta h} = \bar{\rho} + \frac{\bar{K}}{\bar{\rho}}$$

where

$$\bar{K} = \frac{fq^2 M^2}{(7.413x10^{10} D^5)}$$

and is found from a plot of empirical data using

$$D_{v\rho} = (1.4737x10^{-5})\frac{Mq}{D}$$

The pressure drop over short lengths of the well is used to find a pressure profile of the well. The Δp should not be more than 10% of the pressure.

Modified Hagedron & Brown(mH-B)

A two-phase flow correlation using a two phase Reynolds Number. This is for non-bubble flow, Griffith method is used for bubble flow. The input fraction of gas is used to determine if the flow is in the bubble regime, $\lambda_g < L_B$.

$$L_B = 1.071 - .2218\left(\frac{u_m^2}{D}\right) \qquad \lambda_g = {u_{sg}}/{u_m}$$

Then using 4 dimensionless numbers to find the liquid hold up (ψ), and the Reynolds number from the correlation graphs. Using the two-phase Reynolds number and the Moody chart to find the friction factor to use in the equation.

$$144\frac{dp}{dh} = \bar{\rho} + \frac{fw^2}{\left(7.413E10D^5\right)\bar{\rho}}$$

If the flow is in the bubble flow regime use the Griffith correlation.

$$144\frac{dp}{dh} = \bar{\rho} + \frac{fw_l^2}{(7.413E10)D^5\rho_l y_l^2}$$

$$y_l = 1 - \frac{1}{2}\left[1 + \frac{u_m}{u_s} - \sqrt{\left(1 + \frac{u_m}{u_s}\right)^2 - 4\frac{u_{sg}}{u_s}}\right]$$

The Reynolds number is calculated by the equations

$$N_{\text{Re}} = \frac{D\bar{u}_l\,\rho_l}{\mu_l} \qquad N_{\text{Re}} = \frac{2.2E - 2w_l}{D\mu_l}$$

Gilbert Method

A graphic method using plots for each flow rate, tubing size, oil and gas gravities and temperature. These are plots of pressure vs length, by using equivalent length either the bottom hole pressure can be found from the tubing head pressure, or the tubing head from the bottom hole pressure.