**Stimulation**

- Bottomhole frac pressure = surface pressure + hydrostatic – friction terms

\[ p_f = p_w + p_h - p_{tf} - p_{pf} \]

- Fracture gradient

\[ F_G = \frac{p_w + p_h - p_{tf} - p_{pf}}{D} \]

- Instantaneous shutin pressure

\[ P_{isip\text{(at surface)}} = p_f - p_h = F_G(D) - p_h \]
Fracture down tubing with packer

• Why?
  – Integrity of casing
  – Higher burst rating,
    2 7/8”, 6.5#, N-80 → $p_{\text{burst}} = 10,570$ psi
    5 ½ ”, 15.5#, J-55 → $p_{\text{burst}} = 5,320$ psi

• Disadvantage:
  Higher frictional effects – important in surface injection pressures and allowable rates

• Considerations: burst of tubing, force and length changes on packer and tubing.
Fracture down casing

Why?

- Allows higher injection rates (>25 bpm) at lower surface injection pressures

• **Disadvantage:** burst rating of casing, integrity

• **Considerations:** burst of casing at breakdown and screenout
Fracture treatment with live annulus (*Preferred)

• How?
  - Pump through tubing, monitor bottomhole pressure through annulus
    - Annulus full of fluid of known density
    - Measure surface annulus pressure

• Considerations:
  Burst of tubing
Fracture treatment with live tubing (*Preferred)

• How?
  • Pump through annulus, monitor bottomhole pressure through tubing
    – tubing full of fluid of known density
    – Measure surface tubing pressure

• Considerations:
  collapse of tubing, burst of casing
Hydrostatic pressure

- Fundamental equation:
  \[ p_h = \frac{z_2}{z_1} \int \rho gdz \Rightarrow \rho gh \]

- In field units: \( p_h = 0.052 \rho_f h \) \{psi\}

- Where \( h \) is TVD in ft and \( \rho_f \) is fluid/slurry density

- Calculate density from:
  \[ \rho_f = \frac{8.34 \gamma_f + x}{1 + \frac{x}{8.34 \gamma_p}} \]
Hydrostatic pressure

Calculate density from:

\[ \rho_f = \frac{8.34 \gamma_f + x}{1 + \frac{x}{8.34 \gamma_p}} \]

- \( \rho_f \) - density of frac fluid [ppg]
- \( \gamma_f \) - specific gravity of frac fluid
  - Fresh water, 8.34 ppg, \( \gamma_f = 1.00 \)
  - 2% Kcl water, 8.43 ppg, \( \gamma_f = 1.01 \)
  - 43 API oil, 6.76 ppg, \( \gamma_f = 0.81 \)

- \( \gamma_p \) - specific gravity of propping agent
  - sand 2.65
  - resin-coated sand 2.55
  - ceramic proppants 2.7 to 3.3
  - bauxite \( \geq 3.4 \)

- \( x \) - concentration of propping agent [ppg]
Hydrostatic pressure

<table>
<thead>
<tr>
<th>Sp. Gr. at 60°F</th>
<th>Lbs. Per Gal. at 60°F</th>
<th>Lbs. Per Cu. Ft. at 60°F</th>
<th>KC1 Lbs.</th>
<th>Fresh Water Gals.</th>
<th>Freezing Point °F</th>
<th>PSI Per Ft. Depth</th>
<th>Approximate % KC1</th>
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</tbody>
</table>
Stimulation

Methods to obtain

1. Use provided friction charts from service company. Good for non-newtonian fluids and specialized systems.

2. From field measurements
   - includes all friction terms, i.e., perf, tubulars, fracture
   - shutdown during pad to avoid proppant effects
   - use to calibrate friction terms

3. Calculate friction
   - Newtonian vs non-Newtonian
   - Laminar vs turbulent
   - Reynolds Number – friction factors

\[
\Delta p_w = p_{fric} \\
p_{w1} = p_f - p_h + p_{fric} \\
p_{w2} = p_f - p_h
\]
Example

**Well Data**

Depth: 6,650 ft
ISIP data on offset wells:
Well 1: $p_{\text{isip}} = 1,870$ psi with proppant-free fresh water in hole
Well 2: $p_{\text{isip}} = 2,300$ psi with proppant-free 36° API oil in hole

41/2 -in. casing set on top of pay zone.
Working pressure limit: 3,900 psi on casing

The breakdown pressure is within the 3,900 psi casing working pressure limit.

**Determine:**

Hydraulic horsepower required for treatment of 25 bbl/min using un-treated fresh water with 1 lb/gal sand.
Injection rate and hhp for treatment at maximum rate without exceeding casing working pressure while pumping untreated fresh water with no sand.
**Stimulation**

**Solution**

Calculate the surface injection pressure required to inject sand-laden water at a rate of 25 bbl/min. Determine the bottom-hole treating pressure $p_f$ from offset well ISIP data.

**Well 1**  
$p_f = p_{isip} + p_h$  
$= 1870 + 0.433 \times 6650$  
$= 4,750 \text{ psi}$

**Well 2**  
$p_f = p_{isip} + p_h$  
$= 2300 + 0.365 \times 6650$  
$= 4,730 \text{ psi}$

Obtain the surface injection pressure from

$$p_w = p_f - p_h + p_{tf} + p_{pf}$$

Where the $p_{pf}$ term becomes zero since the well is an open-hole completion and $p_{tf}$ is known from offset well data. The pipe friction can be calculated using friction-loss vs injection-rate curves.
Stimulation

Solution

For water displaced through 4 1/2 -in. OD casing the friction pressure is 200 psi/1,000 ft of casing while pumping water at 25 bbl/ min. However, this friction pressure must also be adjusted for the effect of sand concentration. The correction factor is 1.10 for water containing 1 lb of sand/gal (see Figure 1). The hydrostatic pressure of fresh water containing 1 lb/gal sand is 465 psi/ 1,000 ft of hole. Hence:

$$ p_{tf} = \frac{200 \text{ psi}}{1000 \text{ ft}} \times 6650 \text{ ft} \times 1.10 = 1463 \text{ psi} $$

$$ p_{h} = \frac{465 \text{ psi}}{1000 \text{ ft}} \times 6650 \text{ ft} = 3090 \text{ ft} $$

Figure 1. Effect of Sand Concentration on friction pressure

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Stimulation

Solution

\[ p_w = p_f - p_h + p_{tf} + p_{pf} \]
\[ = 4740 - 3090 + 1463 + 0 \]
\[ = 3113 \text{ psi} \]

Thus a surface pressure of 3,113 psi is required to inject fresh water containing 1 lb of sand/gal down the 4 1/2-in. casing at a 25 bbl/min rate.

Determine the hydraulic horsepower required, using the following equation.

\[ hhp = 0.0245Qp_w = 0.025 \times 25 \times 3113 = 1906 \]

To calculate the maximum injection rate possible without exceeding a surface pressure of 3,900 psi, determine first the amount of friction pressure that may be expended,

\[ p_{tf} = p_w + p_h - p_f - p_{pf} \]
\[ = 3900 + 3090 - 4740 - 0 \]
\[ = 2250 \text{ psi} \]
Solution

Convert the total allowable friction to friction loss per 1000 ft of pipe,

\[
\frac{2250 \text{ psi}}{6.65 \times 1000} = 353 \text{ psi/1000 ft}
\]

The maximum pump rate may be determined by using the 4 ½ inch casing curve for water. The injection rate corresponding to a friction loss of 353 psi/1000 ft is 35 bpm. The hhp necessary to frac at 35 bpm with a surface pressure of 3,900 psi is 3,344.
Perforation friction pressure

- Important in fracture design and pressure analysis
- Semi-empirical model
  based on similarity to orifice equation
  \[ f(\text{flow rate, slurry density, perforation density and number}) \]

\[
\Delta p_{pf} = \frac{0.2369 q^2 \rho_f}{2 d_p^4 k_d^{1/2} n_p^2}
\]

where,
- \( q \) = flow rate, bpm
- \( \rho_f \) = fracture fluid density, ppg
- \( d_p \) = perforation diameter, in.
- \( n_p \) = perforation number
- \( k_d \) = discharge coefficient, measure of the perfs efficiency at passing fluid

\( k_d \approx 0.60 \) for new perfs
\( k_d \approx 0.85 \) for eroded perfs
Perforation friction pressure
Perforation friction pressure

- Time-dependent perforation friction

Diversion techniques for multiple zones

1. Packer and bridge plug arrangement
2. Multistage w/ ball sealers
3. Baffles
4. Sand plugs
5. Limited Entry
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**Limited Entry**

**Definition**
Number of perfs intentionally limited to cause high downhole pressures which result in simultaneous stimulation of zones with different closure pressure.

**Discussion**
Limited entry is a technique used in fracturing or acidizing to help control fluid entry into the formation through a predetermined number of perforations. The number and size of these perforations will depend upon the formation bottom hole treating pressure, type of fluid being used, size of conductor pipe through which the treatment is to be performed and surface limitations. These perforations can be placed in the well so that a desired amount of treatment volume can be injected into any zone.
Limited Entry

Example

• Given: Condition of well and treating fluid

Pipe Size - 4 1/2 inch 11.6 lb casing
Depth - 6,000 ft
BHTP - 2000 psi
Fluid type - 2% potassium chloride brine
Viscosity - 1 cp
Average sand concentration - 1 lb/gal
Perforation diameter - 0.5 in.
Desired perforation friction ($p_{pf}$) - 600 psi
Injection rate (Q) - 40 BPM
Assumed perforation coefficient - 0.95

• Determine: Wellhead pressure, hydraulic horsepower, and number of 0.5 in. perforations required
Limited Entry

Solution:

\[ p_w = 2000 - p_h + p_{tf} + 600 \]

\[ = 2600 - p_h + p_{tf} \]

Hydrostatic pressure:

Pressure gradient for 2% KC1 water with 1 ppg = 47.0 psi/100 ft

\[ p_h = (47)(60) = 2820 \text{ psi} \]

Friction pressure:

Friction loss for a 1 cp fluid in 4 1/2 inch, 11.6 lb casing at 40 BPM= 52 psi/100 ft

\[ p_{tf} = (52)(60) = 3120 \text{ psi} \]

Therefore:

Wellhead pressure: \[ p_w = 2600 - p_h + p_{tf} = 2900 \text{ psi} \]

Hydraulic horsepower:

\[ \frac{Q * p_w}{40.81} = 2842 \text{ hp} \]

Number of perforations:

\[ \rho_f = \frac{8.34 \gamma_f + x}{1 + \frac{x}{8.34 \gamma_p}} = \frac{8.34*1.01+1}{1+\frac{1}{8.34*2.65}} = 9.016 \text{ ppg} \]
Solution:

\[
\left( \frac{q}{n_p} \right)^2 = \frac{d^4 k^2 d \Delta p pf}{.2369 \rho_f} = \frac{.5^4 \cdot .90^2 \cdot 600}{.2369 \cdot 9.016} = 14.44
\]

Rate per perforation was found to be 3.8 BPM per perforation.

\[
n_p = \frac{q}{3.8} = 10.5 \text{ or } 11 \text{ perforations}
\]

This limited entry solution would require 11 perforations 0.5 inch in diameter open and accepting fluid to treat this well at 40 BPM with a wellhead pressure of 2900 psi using 2842 hydraulic horsepower.

Consequences:
Too many perfs, velocity ↓, thus proppant could settle...screenout.
Too few perfs, restrict overall injectivity, HHP↑, cost ↑