Porosity – Permeability Relationships

Permeability and porosity trends for various rock types [CoreLab, 1983]
Porosity – Permeability Relationships

Influence of grain size on the relationship between porosity and permeability
[Tiab & Donaldson, 1996]
Porosity – Permeability Relationships

- **Darcy’s Law (1856)** – empirical observations of flow to obtain permeability

- **Slichter (1899)** – theoretical analysis of fluid flow in packed uniform spheres

- **Kozeny (1927), Carmen (1939)** – capillary tube model
Porosity – Permeability Relationships

**Capillary Tube Model**

Define porosity → \( \phi = n_t \pi r^2 \sqrt{\tau} \)

Where \( r \) is radius of the capillary tube, \( n_t \) is number of tubes/ unit area

Define permeability → \( k = \frac{\pi n_t r}{8 \sqrt{\tau}} \)

Porosity-permeability relationship → \( k = \frac{\phi r^2}{8 \tau} \)
Porosity – Permeability Relationships

Example

For cubic packing shown, find $\phi$ and $k$.

Number of tubes per unit area: 4 tubes$/ (4r)^2$

Porosity
\[ \phi = \frac{1}{2} \times \frac{\pi r^2}{4r^2} = \frac{\pi}{4} \]

Tortuosity
\[ \tau = \left( \frac{L_a}{L} \right)^2 = 1 \]

Permeability
\[ k = \frac{\phi r^2}{8\tau} = \frac{\pi}{4} \times \frac{r^2}{8(1)} = \frac{\pi r^2}{32} \]
Porosity – Permeability Relationships

Define specific surface area

\[
S_{pv} = \text{specific surface area per unit pore volume} \\
S_{pv} = 2/r \text{ (for cylindrical pore shape)}
\]

\[
S_{bv} = \text{...unit bulk volume}
\]

\[
S_{gv} = \text{...unit grain volume}
\]

\[
S_{bv} = \phi \times S_{pv}
\]

\[
S_{gv} = \left(\frac{\phi}{1-\phi}\right) S_{pv}
\]

Carmen – Kozeny Equation

\[
k = \frac{\phi r^2}{8\tau}
\]

\[
S_{pv} = 2/r
\]

Where

\[
k = \frac{\phi}{k_z S_{pv}^2}
\]

Kz, Kozeny constant-shape factor to account for variability in shape and length
**Porosity – Permeability Relationships**

### Carmen – Kozeny Equation

\[
k = \frac{\phi r^2}{8\tau} \quad S_{pv} = 2/r
\]

\[
k = \frac{\phi}{k_z S_{pv}^2}
\]

Where

- **Kz**, Kozeny constant-shape factor to account for variability in shape and length

**k₀** is a shape factor
- = 2 for circular
- = 1.78 for square

### Carmen – Kozeny Equation

\[
k_z = k_o \times \tau
\]

Tortuosity, \( \tau \)

\[
\tau = \left( \frac{L \alpha}{L} \right)^2
\]

\[
k_o \text{ is a shape factor}
\]

- = 2 for circular
- = 1.78 for square
Porosity – Permeability Relationships

Example: spherical particles with diameter, \( d_p \)

\[
k = \frac{\phi}{k_z S_{pv}^2}
\]

\[k = \frac{\phi^3 d_p^2}{72(1-\phi)^2 \tau}\]
Distribution of Rock Properties

Porosity Distribution

Expected porosity histogram

[Amyx, et al., 1960]
Distribution of Rock Properties

Porosity Distribution

Actual porosity histogram
[NBU42W-29, North Burbank Field]
Distribution of Rock Properties

Permeability Distribution

Expected Skewed normal and log normal histograms for permeability
[Craig, 1971]
Distribution of Rock Properties

Permeability Distribution

Actual permeability histogram
[NBU42W-29, North Burbank Field]
Permeability Variation

Dykstra-Parsons Coefficient

\[ V = \frac{k_{50} - k_{84.1}}{k_{50}} \]

Characterization of reservoir heterogeneity by permeability variation

[Willhite, 1986]
Permeability Variation

Example of log normal permeability distribution [Willhite, 1986]
Distribution of Rock Properties

Permeability Variation

Actual Dykstra-Parsons probability plot [NBU42W-29, North Burbank Field]
Permeability Variation

Lorenz Coefficient

\[ L_k = \frac{\text{Area ABCA}}{\text{Area ADCA}} \]

Flow capacity vs storage capacity distribution

[Craig, 1971]
Permeability Variation

Lorenz Coefficient

\[
L_k = \frac{\text{Area ABCA}}{\text{Area ADCA}} = 0.643
\]

Actual Lorenz plot
[NBU42W-29, North Burbank Field]
Distribution of Rock Properties

Drawback of statistical approaches

• Sequential ordering of data

![Schematic of statistical approach of arranging data in comparison to true reservoir data, which is not ordered.](image)

• reliance only on permeability variations for estimating flow in layers. Does not account for:
  – phase mobility, pressure gradient, Swirr and the k/ϕ ratio
Hydraulic Flow Unit

- unique units with similar petrophysical properties that affect flow.
  - Hydraulic quality of a rock is controlled by pore geometry
  - It is the distinction of rock units with similar pore attributes, which leads to the separation of units into similar hydraulic units.
  - not equivalent to a geologic unit. The definition of geologic units or facies are not necessarily the same as the definition of a flow unit.
Distribution of Rock Properties

• Start with CK equation

\[
\sqrt{\frac{k}{\phi}} = \left( \frac{\phi}{1-\phi} \right) \left( \frac{1}{\sqrt{\kappa \tau S_{gv}}} \right)
\]

• Take the log

\[
\log(RQI) = \log(\phi_r) + \log(FZI)
\]

where the Reservoir quality index (RQI) is given by,

\[
RQI(\mu m) = 0.0314 \sqrt{\frac{k_{md}}{\phi}}
\]

the Flow Zone Indicator (FZI) is,

\[
FZI = \frac{1}{S_{gv} \sqrt{k_z}}
\]

and the pore-to-grain volume ratio is expressed as

\[
\phi_r = \frac{\phi}{1-\phi}
\]

Plot of RQI vs $\phi_r$ for East Texas Well
[Amaefule, et al., 1993]
Distribution of Rock Properties

[Diagram showing plots of RQI and Porosity Ratio with data points and lines for various FZI values]

HFU
[NBU42W-29, North Burbank Field]
Distribution of Rock Properties

- Flow units:
  
  \[ y = 578.37e^{-4.647x} \]
  
  \[ R^2 = 0.9917 \]

- Probability of samples with permeability >

- Permeability vs. Porosity:
  
  \[ k = 6E+06 \phi^{6.9644} \]
  
  \[ R^2 = 0.9014 \]

- Frequency vs. Porosity, %

- Porosity Ratio vs. RQI