2.4 Compressibility

Porous rocks when buried at reservoir depths are subject to both internal and external stresses. The internal stress results from the fluid pore pressure. A typical gradient is 0.433 psi/ft. The external stresses are created from the weight of the overburden (typical value is 1.00 psi/ft) and any accompanying tectonic stresses. Tectonic activity such as faulting and folding impart a stress to the rocks. This stress is local in nature or widespread within an entire basin, depending on the geologic history of the area.

The combination of external and internal stresses causes a corresponding strain or deformation in the reservoir rock. External stresses tend to compact the rock and reduce the pore volume. Internal stresses act opposite, therefore resisting pore volume reduction. The difference between the two opposing stresses is known as the net effective stress, $\sigma_e$.

$$\sigma_e = \sigma - p_p$$  \hspace{1cm} (2.17)

Over time, as fluids are produced from the pore space, the pore pressure will decrease. Thus the net effective stress will increase resulting in a porosity reduction with time. Similarly, an increase in the depth of burial will increase the overburden pressure and subsequently reduce the porosity. Figure 2.19 illustrates this concept for two rock types, shale and sandstone.

![Figure 2.19 Effect of natural compaction on porosity [Krumbein & Sloss, 1963]](image)

Classic examples of subsidence are the Ekofisk platform in the North Sea and Long Beach California. The former is due to oil and gas production from highly compressible chalk formation, the latter is due to water withdrawal from a subsurface aquifer. Another application of compressibility is in material balance calculations for undersaturated oils.
The net effective stress relationship has been modified by Biot’s constant, $\alpha$, that measures the effectiveness of the pore pressure in counteracting the total applied load.

$$\sigma_e = \sigma - \alpha p_p$$

(2.18)

The value of $\alpha$ varies between 0 and 1. At the limits when $\alpha = 0$, the pore pressure has no effect on the behavior of the rock, and when $\alpha = 1$ the pore pressure is 100% effective in counteracting the applied load. $\alpha = 1$ is used to evaluate the failure magnitude, while Eq. (2.18) is used to evaluate the deformation of the porous medium. Geertsma and Skempton proposed the following expression for $\alpha$:

$$\alpha = 1 - \frac{K}{K_r} = 1 - \frac{c_m}{c_b}$$

(2.19)

where:

- $c_b$ = bulk compressibility, psi$^{-1}$
- $c_m$ = compressibility of the rock matrix, psi$^{-1}$
- $K$ = effective bulk modulus, psi
- $K_r$ = bulk modulus of the rock solid only, psi

Suklje modified Geertsma's expression by including porosity,

$$\alpha = 1 - (1 - \phi) \frac{K}{K_r}$$

(2.20)

Equations (2.19 and 2.20) are valid only in the ideal case where there is no porosity change under equal variation of pore pressure and confining pressure. A general expression is given by:

$$\alpha = \frac{3(v_u - \nu)}{B(1 - 2\nu)(1 + \nu_u)}$$

(2.21)

where:

- $\nu$ = drained poisson’s ratio
- $v_u$ = undrained Poisson's ratio
- $B$ = Skempton's pore-pressure coefficient.
In liquid-saturated compressible rocks, $B = 1.0$; when the porous space is partially saturated, $B < 1.0$; and $B = 0$ when the rock specimen is dry. Skempton showed that the value of $B$ can be estimated from,

$$B = \frac{1}{(1 + \frac{c_m}{\phi c_f})}$$  \hspace{1cm} (2.21)$$

where $c_m$ and $c_f$ are the compressibilities of the rock and fluids (water) in the void space, respectively.

**Example 2.9**

Knowing the following data:

- $\nu = 0.250$
- $\nu_u = 0.362$
- $E = 2 \times 10^6$ psi
- $c_w = 2.75 \times 10^{-6}$ psi$^{-1}$
- $K_r = 4.83 \times 10^6$ psi
- $\phi = 0.20$

Find the correction factor, $\alpha$.

The bulk modulus of the rock sample is:

$$K = \frac{E}{3(1-2\nu)} = \frac{2 \times 10^6}{2(1-2 \times 0.25)} = 1.33 \times 10^6 \text{ psi}$$

The correction factor $\alpha$,

a. Geertsma and Skempton, $\alpha = 1 - \frac{K}{K_r} = 1 - \frac{1.33 \times 10^6}{4.83 \times 10^6} = 0.72$

b. Suklje, $\alpha = 1 - (1 - \phi) \frac{K}{K_r} = 0.78$
c. General - calculate the rock compressibility and Skempton's pore-pressure coefficient B,

\[ c_m = \frac{1}{K_r} = 0.207 \times 10^{-6} \]

\[ B = \frac{1}{(1 + \frac{c_m}{\phi c_{fl}})} = 0.726 \]

\[ \alpha = \frac{3(v_u - v)}{B(1 - 2v)(1 + v_u)} = 0.68 \]

There is no agreement between the three equations.

To describe the pore and bulk volume variations in petroleum reservoirs, Geertsma (1957) derived expressions relating elastic deformation and fluid pressure decline. In terms of pore volume,

\[ \frac{dV_p}{V_p} = c_m dp + \frac{1}{\phi} (c_b - c_m) d(\sigma - p_p) \quad (2.22) \]

and in terms of bulk volume,

\[ \frac{dV_b}{V_b} = c_m dp + c_b d(\sigma - p_p) \quad (2.23) \]

If the difference between the confining stress, \( \sigma \) and the pore pressure is constant during the triaxial test; i.e., \( d(\sigma - p_p) = 0 \), then Eq. (2.23) reduces to;

\[ c_m = \frac{1}{V_b} \left. \frac{dV_b}{dp} \right|_{\sigma} \quad (2.24) \]

which is defined as the grain or matrix compressibility, \( c_m \), the change in volume of individual rock grains, with respect to a change in pressure at constant temperature. Typically matrix compressibility is small and frequently ignored. Consider the following sequence of steps for a triaxial test. The confining pressure is increased by increments of 200
pis starting at 200 psi and terminating at depth*1 psi/ft. The internal pore pressure is maintained 200 psi less than the confining pressure until reaching the maximum value and then reduced.

<table>
<thead>
<tr>
<th>Step</th>
<th>(P_{ov}), psi</th>
<th>p_e, psi</th>
<th>\sigma_e, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>800</td>
<td>200</td>
</tr>
</tbody>
</table>

\[ d(\sigma - p_p) = 0 \]

\[ d\sigma = 0; d\sigma_e = -dp_p \]

For \( dp_p = -d\sigma_e \), Eq. (2.22) results in an expression for pore volume compressibility, \( c_p \), also known as rock or formation compressibility, the change in pore volume of a formation.

\[ c_p = \frac{1}{V_p} \frac{dV_p}{dp} \left| \frac{1}{\sigma} \right. \]

(2.25)

Pore compressibility ranges from \( 3 - 10 \times 10^{-6} \) for consolidated rocks to \( 30 - 100 \times 10^{-6} \) psi\(^{-1}\) for unconsolidated sands. And last, if only pore pressure is constant \( (dp_p = 0) \), then Eq. (2.23) gives,

\[ c_b = \frac{1}{V_b} \frac{dV_b}{d\sigma_e} \left| \frac{1}{dp_p} \right. \]

(2.26)

the bulk compressibility, \( c_b \), which is the change in bulk volume with respect to pressure.

<table>
<thead>
<tr>
<th>Step</th>
<th>(P_{ov}), psi</th>
<th>p_e, psi</th>
<th>\sigma_e, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>200</td>
<td>800</td>
</tr>
</tbody>
</table>

\[ dp_p = 0 \]

By applying Eq. (2.25), it is possible to derive an expression for pore volume reduction in a reservoir. If we separate and integrate, and assume \( c_p \) is constant over the pressure interval of interest, then,
Equation (2.27) can be simplified to:

\[ V_{p2} = V_{p1} e^{c_p (p_2 - p_1)} \]  

subject to the condition \( c_p \Delta p \ll 1.0 \). Most consolidated reservoir rocks meet this condition.

**Example 2.10**

Calculate the pore volume reduction when the reservoir undergoes production and pressure drops from 5000 to 3000 psi. \( c_p = 3.5 \times 10^{-6} \, \text{psi}^{-1} \).

Applying Eq. (2.26),

\[ \frac{V_{p2}}{V_{p1}} = e^{3.5 \times 10^{-6} (3000-5000)} = 0.993 \]

A 0.7% reduction in volume occurs over a 2000 psi pressure drop.

Equation (2.27) can be expressed in terms of porosity by substituting for \( V_p = \phi \, V_b \).

Therefore,

\[ \phi_2 = \phi_1 e^{c_p (p_2 - p_1)} \]  

As can be seen from Eq. (2.29), as pressure declines, the porosity will decrease. The limitation of the previous relationship is it does not account for a change in bulk volume. An alternative derivation including bulk volume change, results in the following expression for porosity.

\[ \phi_2 = \frac{c_p \Delta p}{e^{c_p \Delta p} - 1} \]  

The assumption to derive Eq. (2.30) is that the grain compressibility is negligible with respect to the change in porosity and therefore is ignored.

**Example 2.11**

For a consolidated sandstone reservoir that has an original porosity of 25%, determine the changes in porosity and thickness for a pore pressure drop of 4000 psi and a \( c_p = 2 \times 10^{-6} \) psi\(^{-1}\) derived from correlations.
Chapter 2 – Porosity

Substitution of variables into Eq. (2.30) results in the following,

\[ \phi_2 = \frac{e^{2 \times 10^{-6} \times (-4000)}}{1 - e^{2 \times 10^{-6} \times (-4000)}} = 24.85\% \]

an absolute porosity change of 0.15% or a relative percent change of 0.6%.

In a reservoir, movement is confined to the vertical direction and therefore a change in thickness can be determined from,

\[ \frac{h_2}{h_1} = 1 - \phi_1 \left[ 1 - e^{cp \Delta p} \right] \]  \hspace{1cm} (2.31)

Substitution of variables results in a thickness ratio of 99.8%; that is, a reduction in thickness by 0.2%. Equation (2.31) is a simplified model to estimate the subsidence phenomena.

Determination of pore compressibility can be accomplished by lab measurements on core samples or by accepted correlations. Two types of loading are applied to core samples in the lab: hydrostatic or triaxial and uniaxial. Figure 2.20 illustrates a triaxial load cell used to measure the changes in pore and bulk volume of a core sample.

![Schematic of a triaxial load cell](image)

Figure 2.20 Schematic of a triaxial load cell [CoreLab, 1983]

Data from the triaxial loading are easily obtained in the laboratory; however the pore compressibility is typically greater than in the reservoir; therefore lab data must be reduced in magnitude prior to application. Uniaxial loading allows only vertical deformation to occur,
while maintaining constant cross-sectional area. These boundary conditions are more representative of those in the reservoir and thus do not require data conversion [CoreLab, 1977].

Figure 2.21 is an example of lab-derived pore compressibilities for an offshore Louisiana, U.S. sandstone. The first compressibility (1) was measured in the lab under triaxial loading conditions and in the last column are the converted pore compressibilities (2) for the difference in loading characteristics. The effective overburden pressure is the difference between the external confining pressure and the internal pore pressure.
### Rock (Pore Volume) Compressibility

**Offshore, Louisiana**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Depth, feet</th>
<th>Permeability to Air, millidarcys</th>
<th>Effective Overburden Pressure, psi</th>
<th>Pore Volume, cm³</th>
<th>Bulk Volume, cm³</th>
<th>Porosity, percent</th>
<th>Compressibility, pV/pV/psi x 10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>99A*</td>
<td>15,639.0-15,640.0</td>
<td>5360</td>
<td>200</td>
<td>6.40</td>
<td>20.20</td>
<td>31.7</td>
<td>27.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td>6.331</td>
<td>20.131</td>
<td>31.45</td>
<td>16.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1000</td>
<td>6.266</td>
<td>20.066</td>
<td>31.23</td>
<td>12.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4000</td>
<td>6.048</td>
<td>19.848</td>
<td>30.47</td>
<td>8.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6000</td>
<td>5.941</td>
<td>19.741</td>
<td>30.09</td>
<td>7.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8000</td>
<td>5.852</td>
<td>19.652</td>
<td>29.78</td>
<td>6.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9000</td>
<td>5.782</td>
<td>19.502</td>
<td>29.53</td>
<td></td>
</tr>
</tbody>
</table>

* Metal sleeve-mounted sample -- special test technique used

1. Measured in laboratory under hydrostatic loading conditions.
2. Uniaxial loading conditions, transformed from hydrostatic data using an average translation factor of 0.61 as per Tew, Dirk: "Prediction of Formation Compaction from Laboratory Compressibility Data," Trans., AIME (1971) 251, 263-271.
Without core samples available for measurement, we rely on correlations to determine compressibilities. In 1953, Hall published the results of laboratory measurements of pore compressibility (Fig. 2.22).

His correlation is reported to be valid for any type of consolidated rock, however it is limited to reservoirs at shallow depths. Van der Knapp (1959) showed that rock compressibility is a function of net effective stress. Figure 2.23 illustrates Van der Knapp’s and Hall’s correlation superimposed on Newman’s (1973) experimental data for limestones. Note that a relationship exists between pore compressibility and porosity; however, there is a wide range in compressibility for a particular porosity.
samples. Figure 2.24 provides the experimental results for sandstones and a correlation for each class is presented in the final graph. Note that for friable and unconsolidated rocks

No relation exists between compressibility and porosity. In summary, realize a wide variation exists of compressibilities for a given porosity. Use lab data if available, otherwise Newman’s correlation is recommended over the preceding two methods.

Example 2.12
Repeat Example 2.11 for an unconsolidated rock. Using Newman’s correlation and find the reduction in porosity and thickness. Compare with the results from Example 2.11.
Solution

From Figure 2.24, for unconsolidated rock with an initial porosity of 25%, the pore compressibility is $20 \times 10^{-6}$ psi$^{-1}$. Applying Equation (2.28) the porosity reduces to 23.53%, or an absolute porosity change of 1.47% (6% relative change). The thickness ratio becomes 98.1% or a 1.9% reduction. To put this thickness change in perspective, if the original thickness was 100 feet, then the subsidence would be almost 2 ft.

Comparing with the results in Example 2.11 for a consolidated sandstone, the absolute porosity change is minor (0.15%) and the thickness reduction is less (0.2%). As is expected, both porosity and thickness changes greater in unconsolidated vs consolidated rocks.