2.1 Fundamentals

Porosity is one of the most important rock properties in describing porous media. It is defined as the ratio of pore volume to bulk volume of a rock sample.

\[ \phi = \frac{V_p}{V_b} \]  

(2.1)

Even though it is a dimensionless quantity, expressed either in decimal or percentage, it is best to remember that it represents a volume ratio of pore space to the bulk space. Figure 2.1 illustrates a simple example of porosity for a granular media.

It is within these pore spaces that the oil, gas and/or water reside. Therefore a primary application of porosity is to quantify the storage capacity of the rock, and subsequently define the volume of hydrocarbons available to be produced. From a drilling perspective, the rate of penetration and the volume of drilling fluid lost to a formation by invasion are related to porosity. Consider the following example of the effect of porosity on fluid loss.

Example 2.1

For a given fluid loss, differential pressure, mudcake permeability and exposure time indicates that 20 liters of mud filtrate invaded a porous and permeable oil-bearing formation. If the formation thickness, h, is 10 ft., the residual oil saturation is 25%, and the borehole diameter is 8 in., find the invasion diameter if a) porosity is 40% and b) porosity is 10%.

Solution

Assuming piston displacement, the mud filtrate volume is given by,

\[ V_{mf} = \frac{\pi}{4} \left( d_i^2 - d_h^2 \right) h\phi S_{xo} \]

Rearranging and substituting values,
Porosity is an average property defined over the representative elementary volume (REV) [Bear, 1972]. That is, as shown in Figure 2.2, the porous media volume must be larger than the size of a single pore so that adding more pores will maintain a meaningful statistical average; but must be smaller than the heterogeneity of the entire flow domain. Thus the REV provides a uniform porosity value for the domain of the porous media.

Several types of porosity have been defined based on the degree of connectivity or the time of pore development. Total porosity is the ratio of the total pore space of the media to the total bulk volume. Effective porosity is the ratio of interconnected pore space to the bulk volume of the rock. Figure 2.3 is an example of total vs. effective porosity in a vuggy rock. Notice the pathway for fluid to migrate in connected pores and the isolated nature of others. Production of hydrocarbons is dependent upon the fluid to flow in the porous media.
Subsequently, it is the effective porosity, which is of importance to reservoir engineering. Isolated pores are of little value to the recovery of hydrocarbons.

Figure 2.3 Isolated vs. connected pores in a vuggy rock. [Asquith, 1985]

Another definition of effective porosity, widely accepted in log interpretation, is the interconnected, clean pore space. Figure 2.4 illustrates the three types of clay distributions found in rocks and the effect on porosity. Dispersed shale resides in the pore space and substantially reduces the total porosity to:

\[
\phi_e = \phi_i - V_{sh}
\]  

where \( V_{sh} \) is the shale content, defined as the silt, clay and bound water components.

Figure 2.4 Shale distributions and effect on porosity in sediments [Dewan, 1983]
Laminated shale is alternating layers of shale and sand and therefore occupies both the pore space and matrix space. Effective porosity is defined as:

$$\phi_e = \phi_i - V_{sh} \phi_{sh}$$  \hspace{1cm} (2.3)

Recent investigations on dispersed sand/shale sequences have become popular for studies of elastic properties for various sand/shale mixtures [Dvorkin & Gutierrez, 2002]. Figure 2.5 illustrates the variations from pure sand (left side of figure) with porosity $\phi_{ss}$ and grain radius, $R$, to pure shale (right side of figure) with porosity $\phi_{sh}$ and let the shale platelets be approximated by isotropic spheres of radius $r$, where $r \ll R$.

Depending on the relative concentrations of the large and small grains, various mixture configurations are possible. The *critical concentration* is the point where small grains completely fill the pore space of the large grain pack while the large grains are still in contact with each other (middle figure). This point indicates the separation between two structural domains. The domain on the left is where an external load is supported by the large grain framework, hence it is a shaly sand. In the domain on the right the large grains are suspended in the small particle framework which is load bearing; e.g., sandy shale.

The total porosity for the shaly sand (where $\beta \leq \phi_{ss}$) is

$$\phi = \phi_{ss} - \beta \phi_{sh}$$  \hspace{1cm} (2.4)

where

$$\beta = \frac{r^3 * n_s}{R^3 * n_L / 1 - \phi_{sh}}$$  \hspace{1cm} (2.5)
is the volume fraction of shale in the entire rock and \( n_s \) and \( n_L \) are the number of small and large grains in the mixture, respectively.

For the sandy shale (where \( \beta > \phi_s \)), total porosity is given by:

\[
\phi = \phi_{sh} \times C \tag{2.6}
\]

where

\[
C = \frac{1}{1 + \frac{1 - \phi_{ss}}{\beta}} \tag{2.7}
\]

In this case, the volume fraction of the small grain pack in the mixture (C) is equivalent to the volume fraction of shale in the whole rock.

The total porosity for the laminated sand/shale sequence is simply the weighted average of the sand and shale porosities.

\[
\phi = \omega \phi_{sh} + (1 - \omega) \phi_{ss} \tag{2.8}
\]

where the weighting parameter is defined as,

\[
\omega = \frac{\beta}{\beta + 1} \tag{2.9}
\]

*Primary porosity* develops during the deposition of the sediments; i.e., intergranular or intercrystalline. *Secondary porosity* is developed by a diagenetic process subsequent to the deposition. Fractures and vugs are classic examples of secondary porosity. Figure 2.6 is an example of types of porosity in sandstone reservoirs. The primary porosity is the intergranular, formed during deposition. The fractures and dissolution are types of secondary porosity.

Figure 2.6 Types of porosity in sandstone reservoirs [Tiab & Donaldson, 1996]
Sediments at the surface typically exhibit high porosities. Progressive burial and compaction will decrease the porosity as a function of depth. In a continuous depositional environment for Tertiary-age rocks, an exponential decrease is expected [Hilchie, 1978],

\[ \phi = \phi_o e^{-cD} \]  

(2.10)

where \( \phi \) is the porosity at depth D, \( \phi_o \) is the surface porosity, and c is an empirical constant. Typically, shale porosities exhibit the largest variance in porosity from the surface and thus are commonly used to estimate pore pressure of reservoirs.

2.2 Geologic Aspects

From the previous section it should be evident that geologic processes play a significant role in porosity development. Therefore, this section will describe the parameters which influence porosity; for example grain size and sorting or dissolution. Sediments can be divided into two general classes: clastics and carbonates. Both have similarities and differences in porosity development and therefore will be discussed separately.

Clastics

Clastics define a group of sediments which are granular in nature. For example sandstones and siltstones are the common type of sediments known in this group. They are typically separated by grain size as shown in Figure 2.7. For example, a grain diameter of 1/16 to 2 mm constitutes a sand, from 1/256 to 1/16 mm is siltstone, and < 1/256 is considered clay.

Figure 2.7 Range of particle sizes in clastic sediments [Krumbein & Sloss, 1963]
A second textural parameter important for describing sediments is the grain size distribution or also known as sorting. Figure 2.8 illustrates the difference between excellent and poor sorting of sand grains. The top picture shows excellent sorting for both coarse and fine-grained rocks. The bottom two illustrations show a mixing of grain sizes or a distribution of grain sizes in the rocks.

![Figure 2.8 Examples of sorting of grains](image)

Grain size distributions are typically presented in histograms or cumulative frequency plots. Figure 2.9 shows a set of four histograms from clean dune sand to shale with the accompanying frequency plots.
Example 2.2

In unconsolidated formations, sand control is a required component for a successful gravel pack completion. The selection of the gravel size is determined by the particle size of the formation sand and is approximately 5 to 6 times the geometric average of the formation sand [Golan & Whitson, 1991]. Too large of a gravel and formation fines will migrate and plug the completion; too small of a gravel and a flow restriction will develop. Sieve analysis of the formation sand is given in Table 2.1. Plot the cumulative weight percent vs. the particle size diameter and determine the mean formation grain diameter. From Table 2.2, what gravel size would you select for this well?
CHAPTER 2 – Porosity

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Grain dia., in</th>
<th>Weight retained (gm)</th>
<th>Weight %</th>
<th>Cumulative weight, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>.0930</td>
<td>0.25</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>.0661</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>.0469</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>.0331</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>.0232</td>
<td>0.79</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>.0165</td>
<td>2.81</td>
<td>15.4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>.0117</td>
<td>3.25</td>
<td>17.8</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>.0059</td>
<td>4.10</td>
<td>22.5</td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>.0041</td>
<td>4.52</td>
<td>24.8</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>.0029</td>
<td>2.52</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>.0021</td>
<td>18.24</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>325</td>
<td>.0017</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pan</td>
<td>2.52</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Sieve Analysis results for example 2.2

<table>
<thead>
<tr>
<th>U.S. Mesh size</th>
<th>Minimum grain dia., in.</th>
<th>Maximum grain dia., in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>40/100</td>
<td>0.006</td>
<td>0.017</td>
</tr>
<tr>
<td>40/70</td>
<td>0.008</td>
<td>0.017</td>
</tr>
<tr>
<td>40/60</td>
<td>0.010</td>
<td>0.017</td>
</tr>
<tr>
<td>20/40</td>
<td>0.017</td>
<td>0.033</td>
</tr>
<tr>
<td>16/30</td>
<td>0.023</td>
<td>0.047</td>
</tr>
<tr>
<td>12/20</td>
<td>0.033</td>
<td>0.066</td>
</tr>
<tr>
<td>12/18</td>
<td>0.039</td>
<td>0.066</td>
</tr>
<tr>
<td>10/20</td>
<td>0.033</td>
<td>0.079</td>
</tr>
<tr>
<td>10/16</td>
<td>0.047</td>
<td>0.079</td>
</tr>
<tr>
<td>8/12</td>
<td>0.066</td>
<td>0.094</td>
</tr>
<tr>
<td>6/10</td>
<td>0.079</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Table 2.2 Commercial gravel data

Solution

The sieve analysis data is plotted on the accompanying log-probability graph to determine the mean grain diameter and distribution. From the straight line relationship a log-normal distribution is evident; therefore the 50% cumulative weight percent represents the geometric mean grain size, $d_r = 0.0036$ in. Using the criteria for gravel pack size of 5 to 6 times the formation grain size, results in 0.018 to 0.022 in. Based on this range, the 20/40mesh sand in table 2.2 is the recommended gravel size.
The size of the grains has little, if any, effect on the porosity; however, the degree of sorting has a significant impact as shown in Figure 2.10. Clean sands of various sizes were artificially mixed in the laboratory and then porosity measured. The results show for all grain sizes from coarse to very fine sand, the porosity increases as the sorting improves from poor to extremely well sorted. Second, notice that porosity is uniform for a given degree of sorting and various grain sizes. That is, porosity does not change (within experimentation error) irregardless of the size of the grains.

Figure 2.10 Effect of grain size and sorting on porosity
Another textural parameter of importance is the packing or arrangement of grains. As shown in Figure 2.11, for uniform grains the porosity will be different for cubic vs. rhombohedral structures, with the cubic packing the maximum for uncompacted sand grains.

![Figure 2.11 Example of cubic and rhombohedral packing [Amyx, et al., 1960](image)](image)

**Example 2.3**

Determine the porosity for the cubic packing arrangement in Figure 2.11.

**Solution**

Define the unit cell with sides equal to twice the radius of the sand grain, 2r. The bulk volume of the cell becomes,

\[ V_b = \pi r^3 = 8r^3 \]

The volume of an individual sand grain is, \( V_g = \frac{4}{3} \pi r^3 \). Within the unit cell there are 8 – 1/8 sand grain spheres, or one grain volume. Porosity can now be determined from Eq. (2.1),

\[ \phi = \frac{V_b - V_g}{V_b} = \frac{3}{8r^3} - \frac{4/3\pi r^3}{8r^3} = 1 - \frac{\pi}{6} = 47.6\% \]

Notice the radius of the sand grains cancel and therefore porosity is a function of packing only.
The degree of sorting and packing are dependent upon two final textural parameters; roundness and fabric of the grains. The degree of roundness and sphericity is a function of the erosional processes the grains have undergone during transport and burial. Figure 2.12 presents images of varying roundness and sphericity.

![Figure 2.12 Roundness and sphericity](Pettijohn, et al., 1973)

The sphericity of a grain refers to the dimensions of the grain with respect to an equidimensional sphere. The roundness is related to the roughness of the grain from angular to smooth. The fabric is the composite effect or framework of the various grains. Figure 2.13 is a schematic illustrating the various contacts and Figure 2.14 is an example thin section of grain and pore configuration.

![Figure 2.13 Sketch of fabric styles](Pettijohn, et al., 1973)
CHAPTER 2 – Porosity

Figure 2.14 Thin section illustrating grain and pore configuration [Amyx, et al., 1960]

Figure 2.14b Thin section from the Queen Sand Formation, Round Tank Field, SE NM, to determine the minerals, clays, depositional environment, and general structure. The thin sections were stained blue to show the porosity and yellow for potassium feldspar. In the pictures, the bright, multicolored areas are anhydrite. The field of view for this picture is 80 microns and the average grain size is estimated to be less than 10 microns. The amount of blue indicates fairly high porosity, with much of it being secondary from the dissolution of feldspars and cements. Some of the cements could have been gypsum, which would have been a precursor to the anhydrite cement. Figure 2.14b shows fine grained sandstone and as permeability is a function of grain size squared, this partially explains the low permeability exhibited by the rock.
CHAPTER 2 – Porosity

Carbonates

Carbonate rocks will sometimes exhibit similar porosity traits as defined previously for clastics; e.g. intergranular. However, they are more susceptible to post-depositional chemical dissolution and therefore are dominated by secondary porosity development. Typical carbonates are composed of two components, the finer -grained matrix material and the allochems. The interstitial matrix material is known as micrite if it is very fine, subcrystalline texture, or sparite if it is coarsely crystalline, fine textured. Allochems consists of fossils, molds, oolites, or intraclasts. These are predominantly large fragments or aggregates floating within the matrix.

Figure 2.15 shows the common types of limestone porosity from vuggy (1) to primary (2), (3) to fractures (4). Note the heterogeneous and random nature of the pore space.

The evolution of vuggy porosity is illustrated in Figure 2.16. The progressive etching of the carbonate material enlarges a mold into a vug. Later migration of mineral rich waters can partially or completely infill the vugs. The result is a reduction or complete loss of the secondary porosity.

Comparison of porosity in sandstone and carbonate rocks is detailed in Table 2.3. In summary, porosity in sandstones is dominated by textural parameters developed during the
deposition of the sediments. On the other hand, carbonate porosity is typically secondary in nature; i.e., fracturing or dissolution channels or vugs.

Figure 2.16 Evolution of moldic (vuggy) porosity [Link, 1982]
<table>
<thead>
<tr>
<th>Aspect</th>
<th>Sandstone</th>
<th>Carbonate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of primary porosity in sediments</td>
<td>Commonly 25-40%</td>
<td>Commonly 40-70%</td>
</tr>
<tr>
<td>Amount of ultimate porosity in rocks</td>
<td>Commonly half or more of initial Porosity, 15-30% common</td>
<td>Commonly none or only small fraction of initial porosity; 5-15% common in reservoir facies</td>
</tr>
<tr>
<td>Types of primary porosity</td>
<td>Almost exclusively interparticle</td>
<td>Interparticle commonly predominates, but intraparticle and other types are important</td>
</tr>
<tr>
<td>Types of ultimate porosity</td>
<td>Almost exclusively primary interparticle</td>
<td>Widely varied because of post-depositional modifications</td>
</tr>
<tr>
<td>Size of pores</td>
<td>Diameter and throat sizes closely related to sedimentary particle size and sorting</td>
<td>Diameter and throat sizes commonly show little relation to sedimentary particle size or sorting</td>
</tr>
<tr>
<td>Shapes of pores</td>
<td>Strong dependence on particle shape - a &quot;negative&quot; of particles</td>
<td>Greatly varied, ranges from strongly dependent “positive” or “negative” of particles to form completely independent of shapes of depositional or diagenetic components</td>
</tr>
<tr>
<td>Uniformity of size, shape, and distribution</td>
<td>Commonly fairly uniform within homogeneous body</td>
<td>Variable, ranging from fairly uniform to extremely heterogeneous, even within body made up of single rock type</td>
</tr>
<tr>
<td>Influence of diagenesis</td>
<td>Minor; usually minor reduction of primary porosity by compaction and cementation</td>
<td>Major; can create, obliterate, or completely modify porosity; cementation and solution important</td>
</tr>
<tr>
<td>Influence of fracturing</td>
<td>Generally not of major importance in reservoir properties</td>
<td>Of major importance in reservoir properties if present</td>
</tr>
<tr>
<td>Visual evaluation of porosity and permeability</td>
<td>semiquantitative visual estimates commonly relatively easy</td>
<td>Variable; semiquantitative visual estimates range from easy to virtually impossible; instrument measurements of porosity, permeability and capillary pressure commonly needed</td>
</tr>
<tr>
<td>Adequacy of core analysis for reservoir evaluation</td>
<td>Core plugs of 1 in. diameter commonly adequate for &quot;matrix&quot; porosity</td>
<td>Core plugs commonly inadequate; even whole cores (~3-in. diameter) may be inadequate for large pores</td>
</tr>
<tr>
<td>Permeability – porosity interrelations</td>
<td>Relatively consistent; commonly dependent on particle size and sorting</td>
<td>Greatly varied; commonly independent of particle size and sorting</td>
</tr>
</tbody>
</table>

Table 2.3 Comparison of porosity in sandstone and carbonate rocks
Carbonates are usually defined as dual porosity systems, that is, an intergranular or intercrystalline matrix component coupled with a secondary porosity component. Consider a simplified example of a single fracture of width, $w_f$, bisecting two matrix blocks with zero porosity (Figure 2.17).

![Figure 2.17 Schematic of single fracture](image)

Define porosity for this system as:

$$\phi_f = \frac{V_p}{V_b} = \frac{w_f h_f L_f}{AL}$$

(2.11)

If we define the tortuosity, $\tau$ as the square of the flow path length to the unit length and include $n_f$ to represent multiple fractures per unit area, then

$$\phi_f = \frac{(n_f A) w_f h_f}{A} \sqrt{\tau} = n_f w_f h_f \sqrt{\tau}$$

(2.12)

This fracture porosity is the ratio of the fracture pore volume to the unit bulk volume.
CHAPTER 2 – Porosity

If matrix porosity is present, then the total porosity is the sum of the two components.

\[ \phi_t = \phi_f + \phi_m \]  

(2.13)

Selection of samples for core analysis typically skip fractures and thus result in a measurement of matrix porosity. On the other hand, logging tools measure total porosity except for the sonic tool. If two of the three unknowns in Eq. (2.13) are measured the third can be calculated. Pirson in 1970 proposed a partitioning coefficient, \( \nu \), as a measure of the fracture pore volume to the total pore volume of the sample. Expressed in porosity, it is defined as:

\[ \nu = \frac{\phi_f}{\phi_f + \phi_m} \]  

(2.14)

By applying resistivity log measurements Pirson was able to show how to calculate \( \nu \).

Example 2.4
A one-inch cube is bisected by a single fracture with a width of 0.5 mm. What is the fracture porosity? If the fracture density is 2 fractures/unit area, what is the porosity for this cube? If the total porosity of the cube was measured to be 12%, what is the matrix porosity? The partitioning coefficient?

Solution

a. Fracture porosity for a single fracture is given by Eq. (2.11). For simplicity, assume the fracture length is equal to the sample length; therefore tortuosity is equal to one.

\[ \phi_f = \frac{w_f h_f}{A} = \frac{0.5\text{mm} \cdot 1\text{in} \cdot 25.4\text{mm/in}}{1\text{in} \cdot (25.4\text{mm/in})^2} = 2\% \]

b. For multiple fractures, apply Eq. (2.12),

\[ \phi_f = n_f w_f h_f = 2\frac{\text{fractures}}{\text{inch}^2} \cdot \frac{.5\text{mm}}{25.4\text{mm/in}} \cdot 1\text{inch} = 4\% \]

c. The matrix porosity is (Eq.2.13),

\[ \phi_m = \phi_t - \phi_f = .12 - .04 = 8\% \]

d. Partitioning coefficient,

\[ \nu = \frac{4}{4 + 8} = 0.333 \]