Chapter 2
Electrical Properties of Rocks

\[ C_b = \phi^m c_f \times S_w^n \quad M \approx 2.0 \]

Lecture notes for PET 370
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Electrical properties of a rock depend on the pore geometry and fluid distribution.

Electric current by “ionic conduction”

Consider the following tank completely filled with brine water,

- apply a voltage, $v$
- measure a current, $i$
- calculate a resistance by Ohms Law:
  $$ v = i \cdot r $$

Define water resistivity, $R_w$, as:

$$ R_w = r_w \frac{A}{L} $$
• Consider the tank completely filled with 100% brine saturated, porous sand

• Resistance with respect to the water phase

\[ r_w = R_w \frac{L_a}{A_p} \]

• Resistance with respect to fluid-filled, porous rock

\[ r_o = R_o \frac{L}{A} \]

• Since \( r_o \approx r_w \)

\[ \frac{R_o}{R_w} = \left( \frac{A}{A_p} \right) \left( \frac{L_a}{L} \right) \]
• Define the Formation Resistivity Factor, $F$, as:

$$F = \frac{R_o}{R_w}$$

• Define tortuosity;

$$\tau = \left( \frac{L_a}{L} \right)^2$$

• Define porosity,

$$\phi = \left( \frac{A_p}{A} \right)$$

• Thus

$$F = \frac{\sqrt{\tau}}{\phi}$$

*Simplified theoretical relationship between $F$ and $\phi$ does not account for heterogeneity.*
Applications of Formation Factor

1. To determine $R_o$ and subsequently compare with the true formation resistivity, $R_T$, to identify hydrocarbon zones.

2. To determine F and subsequently use to estimate porosity.

3. To determine $R_w$ for water saturation calculations.
Consider a synthetic rock sample made of an insulator material and shaped as a cube of length $L$. There is a square tube of dimension $L/2$ through the cube. Assume the inner square tube is filled with brine of resistivity $R_w$ and that the current will flow perpendicular to the front face.

*Calculate $F$ and the relationship of $F$ with porosity.*
General relationship based on both theoretical and experimental studies is given by:

\[ F = a \phi^{-m} \]

where \( a \) and \( m \) are functions of pore geometry.

**Methods:**

a. Simple theoretical models
   simple models designed with uniform pore geometry do **not** capture variation in porous media.

b. Direct measurement in lab
   accurate but requires rock sample

c. Empirical correlations based on lab data
   most convenient and popular, however may not be appropriate for given rock type
b. Direct measurement in lab

“The practical application of $F = f(\phi)$ for a particular rock type is best accomplished by evaluating the cementation factor using lab-measured values of $F$ and $\phi$.”

....Helander (1983)
Electrical Properties of Rocks

\[ F = \phi^{-m} \]

\( F \) dependent on degree of cementation, thus \( m \) originally defined as “cementation exponent”.

\[ F = \phi^{-m} \]

Archie (1942) suggested the following empirical equation based on lab measurements:

\[ F = \phi^{-m} \]
Empirical Correlations

Winsauer, et al (1952) - analyzed data from 30 samples (28 ss, 1 lms, 1 unconsolidated ss)
Developed correlation known as “Humble Eq.”

\[ F = 0.62\phi^{-2.15} \]

Tixier (1979) – simplified equation using same data

\[ F = 0.81\phi^{-2} \]
Empirical Correlations

Carothers (1968) - analyzed 793 sandstone data points. Generalized correlation:

\[ F = 1.45 \phi^{-1.54} \]
**Humble** - granular or soft rocks, e.g. sandstone

\[ F = 0.62 \phi^{-2.15} \]

**Tixier** - granular or soft rocks, e.g. sandstone

\[ F = 0.81 \phi^{-2} \]

**Archie** - most types of carbonates

\[ F = \phi^{-2} \]

**Shell** - low \( \phi \) (<9%) carbonates, not fractured

\[ F = \phi^{-m} \quad m = 1.87 + 0.019/\phi \]
Define:

- $m$ – pore geometry exponent
- $a$ – pore geometry (tortuosity) factor

Characteristics:

- Coefficient $a$ varies from 0.35 to 4.78 and $m$ from 1.14 to 2.9 (higher in carbonates)
- Observed variation in $m$-exponent, attributed to:
  - Degree of cementation
    - an increase in cementation increases $m$
  - Shape, sorting and packing of grains
  - Types of pores: intergranular, vuggy, fractures
    - fractures $m \sim 1.0$, vugs $m > 2.0$
  - tortuosity
  - constriction in porous network
  - presence of conductive solids
  - compaction due to overburden pressure
  - thermal expansion
Electrical Properties of Rocks

$\phi - F$ relationship

Influence of $a$ and $m$ (Corelab)
Consider the tank filled with a porous sand saturated with both water and hydrocarbons.

Resistance with respect to the water phase only,

\[ r_w = R_w \frac{L'}{A'} \]

Resistance with respect to the porous, hydrocarbon bearing rock,

\[ r_t = R_T \frac{L}{A} \]

Since \( r_t \approx r_w \)

\[ \frac{R_T}{R_o} = \left( \frac{A}{A'} \right) \left( \frac{L'}{L} \right) \]
Electrical Properties of Rocks

Define resistivity index, $I$ as:

$$I = \frac{R_T}{R_o}$$

Archie correlated experimental data and suggested:

$$I \propto \frac{c}{S_w^n}$$

Combine, 

$$\frac{R_T}{R_o} = \frac{c}{S_w^n}$$

Plot,

$$\log(S_w) = \frac{1}{n} \log(c) - \frac{1}{n} \log \left( \frac{R_T}{R_o} \right)$$
Electrical Properties of Rocks

Resistivity-Saturation Relationship

From plot, \( n = 2 \) and \( c = 1 \), thus

\[
\log(S_w) = \frac{1}{n} \log(c) - \frac{1}{n} \log\left( \frac{R_T}{R_o} \right)
\]

From plot, \( n = 2 \) and \( c = 1 \), thus

\[
S_w = \sqrt{\frac{R_o}{R_t}}
\]
Only valid when hydrocarbon and water zones are of the same porosity and salinity

General form known as Archie’s Law.

\[ S_w = \sqrt{\frac{R_o}{R_t}} \]

\[ S_w = \sqrt{\frac{FR_w}{R_t}} \]

*Fundamental relationship which the entire well logging industry is based!!*
Significance of saturation exponent (after Corelab)

\[ RI = \frac{1.0}{S_w^n} \]

- \( n = 2.2 \)
- \( n = 1.6 \)

Effect of “n” on Water Saturation
At RI = 30, \( S_w \) varies from 12% to 21% Pore Space

Resistivity Index vs Water Saturation
For Range of Measured Saturation Exponents
• Observed variation in saturation exponent, $n$ attributed to:

1. **wettability of rock surface**
2. rock texture
3. presence of clays
4. measurement techniques; i.e., static vs dynamic
5. nature of displacing fluid

Fluid distribution in the pore spaces as a function of fluid wettability. Water and oil saturations in (a) a water-wet sand and (b) an oil-wet sand. Pirson (1958)
Wettability influence on rock surface

Resistivity Ratio vs. water saturation in carbonate cores
Anderson, JPT, (Dec 1986)
## Electrical Properties of Rocks

### Exercise

<table>
<thead>
<tr>
<th>Permeable zone indication</th>
<th>Resistivity (ohm-m)</th>
<th>Porosity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>R = 4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>R = 0.4</td>
<td>φ = 0.30</td>
</tr>
<tr>
<td>C</td>
<td>R = 8</td>
<td>φ = 0.07</td>
</tr>
<tr>
<td>D</td>
<td>R = 0.3</td>
<td>φ = 0.35</td>
</tr>
<tr>
<td>Shale</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Notes:
- Resistivity values range from 0 to 10 ohm-m.
- Porosity values range from 0 to 50%.
Chapter 1, Sec 1.1-1.4, 1.6,1.8, Bassiouni, Z: Theory, Measurement, and Interpretation of Well Logs, SPE Textbook Series, Vol. 4, (1994)

Corelab, Fundamentals of Core Analysis, Houston, TX (1983)

