Principle of Superposition

Superposition in space

The principle of superposition states the total pressure drop at any point in the reservoir is the sum of the pressure drops at any point caused by flow in each of the wells in the reservoir. As an example, consider a three well system illustrated in Figure 1.

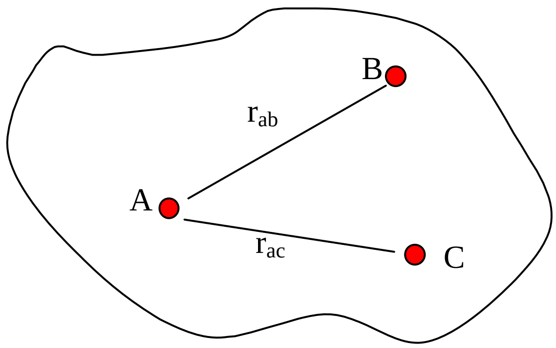


Figure 1. Application of superposition in space for a three well system

The total pressure drop in well A is the summation of the pressure drops caused by each well.

$$\Delta p_{total\ A} = \Delta p_A + \Delta p_B + \Delta p_C \tag{1}$$

Substituting in Eq. (1) the line source solutions, and converting to field units, results in the following equation for total pressure drop in Well A.

$$\begin{aligned} \Delta p_{total\ A} = & \frac{141.2 q_a \mu B_o}{kh}\left[\frac{1}{2}\ln\left(\frac{kt}{1688\phi\mu c_t r_{wA}^2}\right) + S_a\right] \\ & - \frac{141.2 q_b \mu B_o}{kh}\frac{1}{2}Ei\left(-\frac{948\phi\mu c_t r_{ab}^2}{kt}\right) \\ & - \frac{141.2 q_c \mu B_o}{kh}\frac{1}{2}Ei\left(-\frac{948\phi\mu c_t r_{ac}^2}{kt}\right) \end{aligned} \tag{2}$$

Common applications of this method include: evaluating multiwell pressure transient tests; e.g., particularly for interference or pulse tests, simulating pressure behavior in reservoirs with boundaries, and for identifying well locations.

In compact dimensionless form:

$$\Delta p(t,r) = \frac{141.2\mu B}{kh} \sum_{j=1}^{n} q_j P_D(r_{Dj}, t_D) \tag{3}$$

Superposition of variable rate and time

The objective of superposition of variable rate and time is to model variable rate behavior in wells to correctly analyze pressure test measurements. Figure 2 illustrates a simple single well, two-rate test. The problem can be defined as at some time, t, $t > t_1$, what is p_{wf}? The well produces at a rate, q_1 from $t = 0$ to t_1 at which time the production rate changes to q_2 from t_1 to t.

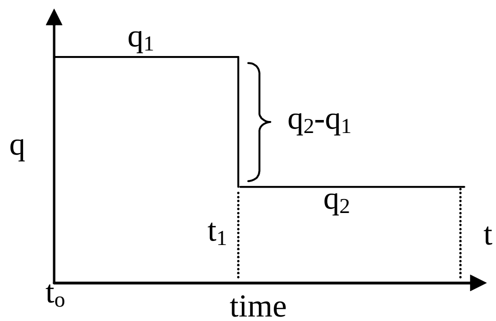


Figure 2. Two-rate test

The solution using the principle of superposition is:

$$\Delta p_T = \frac{141.2 q_1 \mu B}{kh}\left[\frac{1}{2}\ln\left(\frac{k(t-t_o)}{1688\phi\mu c_t r_w^2}\right) + S\right] + \frac{141.2(q_2 - q_1)\mu B}{kh}\left[\frac{1}{2}\ln\left(\frac{k(t-t_1)}{1688\phi\mu c_t r_w^2}\right) + S\right] \tag{4}$$

Expressed in compact form,

$$\Delta p_T = \frac{141.2\mu B}{kh}\sum_{j=1}^{n}\left[\frac{1}{2}\ln\left(\frac{k(t-t_{j-1})}{1688\phi\mu c_t r_w^2}\right) + S\right]\cdot\left(q_j - q_{j-1}\right) \tag{5}$$

The general form of equation for superposition in space and time/rate is given by,

$$\Delta p(t,r) = \frac{141.2\mu B}{kh} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(q_j - q_{j-1} \right) P_D \left[r_{Di}, \left(t - t_{j-1} \right)_D \right] \quad (6)$$

where m is the number of wells and n is the number of rates for each well.

A special case of Eq. (4) occurs if $q_2 = 0$ (shut in). The result is the Horner Approximation to pressure buildup testing.

Method of Images

Employing a technique called the “method of images” derives the pressure behavior of a well near a sealing linear fault or other flow barrier. In this formulation the effect of a fault is simulated by assuming the presence of another identical well producing at a symmetrical position across the fault, as shown in Figure 3. The image well interacts with the actual well so that no flow occurs across the fault. The resulting pressure drop at the real well due to its own production and the "interference drop" from the image well add together to simulate correctly the pressure behavior of the real well as though it were in the proximity of the fault.

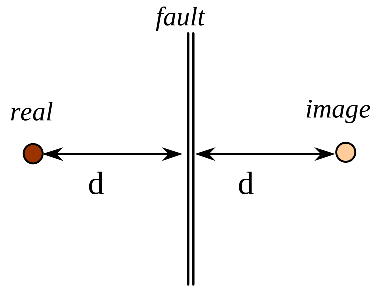


Figure 3. Schematic of a no flow boundary with a real and image well located a distance *d* from the boundary.

By applying superposition we can calculate the pressure at any point in the xy plane. For the special case at the wellbore and constant production rate,

$$P_{wf} = P_i - \frac{141.2 q\mu B}{kh}\frac{1}{2}\left[-Ei(-x)_t^{r_w^2} - Ei(-x)_t^{(2d)^2}\right]$$

or if assume ln approximation, (7)

$$P_{wf} = P_i - \frac{141.2 q\mu B}{kh}\left[\left\{\frac{1}{2}\ln\left(\frac{kt}{1688\phi\mu c_t r_w^2}\right) + S\right\} - \frac{1}{2}Ei\left(-\frac{948\phi\mu c_t (2d)^2}{kt}\right)\right]$$

With the previous definition of x, the second part of Eq. (7) is abbreviated notation where the subscript refers to the time function and the superscript the spatial distance.

Example

Consider a single sealing fault located 250 feet away from the active well. What is the bottom hole flowing pressure after flowing 350 stbd for 8 days, given the following information.

k = 25 md $c_t = 2 \times 10^{-5}$ psia^{-1}

μ = 0.50 cp B_o = 1.13 res bbl/STB

ϕ = 16.0% h = 50 ft

p_i = 3000 psi r_w = 0.333 ft Skin = +5

1. Check if the logarithmic approximation is valid

$$x = \frac{948\phi\mu c_t}{k}\cdot\frac{r^2}{t} = \frac{948(.16)(.5)(2x10^{-5})}{25(8)(24)}r^2 = 3.16x10^{-7} r^2$$

For r = r_w, then $x_{rw} = 3.5x10^{-8}$ << 0.01, thus ln approximation applies

For r = 2L then x_{2L} = 0.079 >> 0.01, thus Ei function applies

2. Calculate the pressure drop

$$\Delta P = \frac{141.2 q\mu B}{kh}\left[\left\{\frac{1}{2}\ln\left(\frac{kt}{1688\phi\mu c_t r_w^2}\right) + S\right\} - \frac{1}{2}Ei\left(-\frac{948\phi\mu c_t (2d)^2}{kt}\right)\right]$$

$$= 22.337\left[\left\{\frac{1}{2}\ln\left(\frac{25(8)(24)}{1688(.16)(.5)(2x10^{-5})(.333)^2}\right) + 5\right\} + \frac{1}{2}(2.039)\right]$$

$$= 22.337 * [(8.2945 + 5) + 1.0195]$$

$$= 297 + 23$$

$$= 320 \text{ psi}$$

3. Calculate the bottom hole flowing pressure

$$p_{wf} = p_i - \Delta p = 3000 - 320 = 2680 \text{ psi}$$

If a pressure buildup test is run in the well then both superposition in space and time must be applied to develop a solution. In abbreviated form, the resulting solution can be expressed as,

$$p_{ws} = p_i - \frac{141.2q\mu B}{kh}\frac{1}{2}\left[-\mathrm{Ei}(-x)^{r_w^2}_{t+\Delta t} + \mathrm{Ei}(-x)^{r_w^2}_{\Delta t} - \mathrm{Ei}(-x)^{(2d)^2}_{t+\Delta t} + \mathrm{Ei}(-x)^{(2d)^2}_{\Delta t} \right] \qquad (8)$$

As an approximation, consider time (t) sufficiently large such that x <<0.01 and thus at the wellbore the logarithmic approximation is reasonable. Also, early in the buildup when Δt is small and provided (d) is large, then the arguments of the Ei function becomes large and thus Ei(∞)→0. Subsequently the last two terms in Eq. (8) approach zero, and the equation reduces to;

$$p_{wf} = p_i - \frac{141.2q\mu B}{kh}\left[\ln\left(\frac{\Delta t + t}{\Delta t}\right)\right] \qquad (9)$$

Thus early in the pressure buildup the slope of the buildup plot will remain unchanged from the conventional analysis. As the shutin time (Δt) increases, the logarithmic approximation becomes reasonable for the image well terms, thus

$$\begin{aligned} p_{wf} &= p_i - \frac{141.2q\mu B}{kh}\left[\ln\left(\frac{\Delta t + t}{\Delta t}\right) + \ln\left(\frac{\Delta t + t}{\Delta t}\right)\right] \\ &= p_i - \frac{141.2q\mu B}{kh}(2)\left[\ln\left(\frac{\Delta t + t}{\Delta t}\right)\right] \end{aligned} \qquad (10)$$

The result is a doubling of the slope. An example is shown in Figure 4.

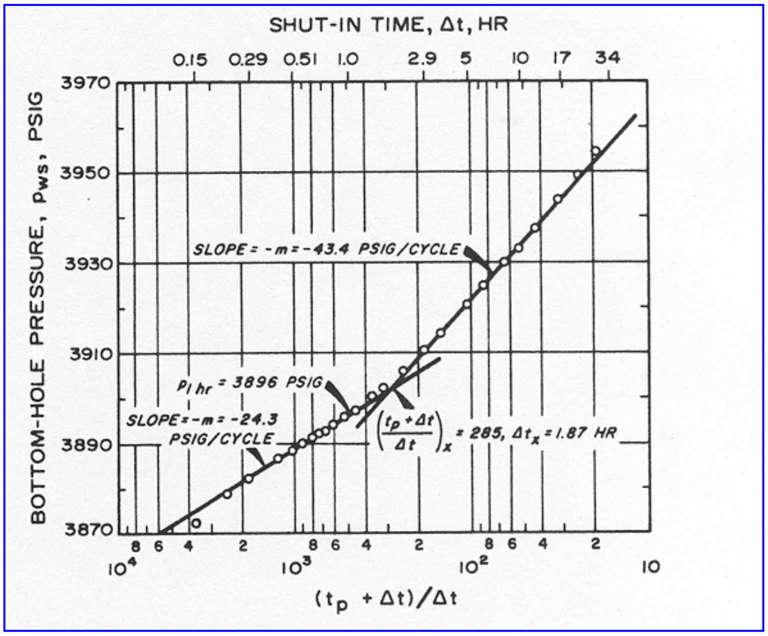
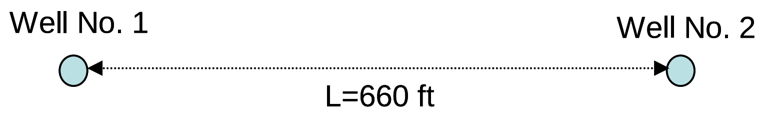
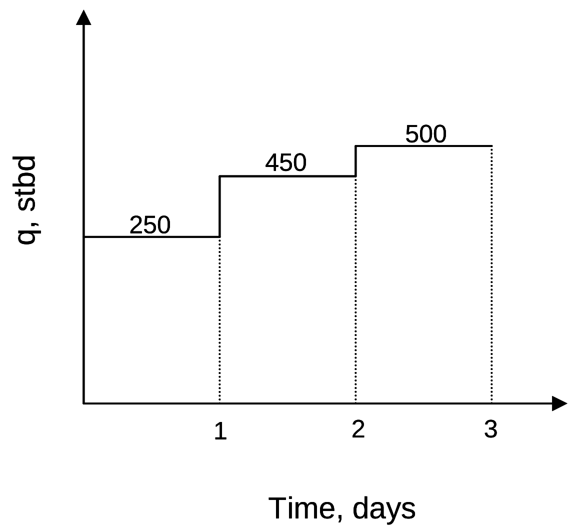


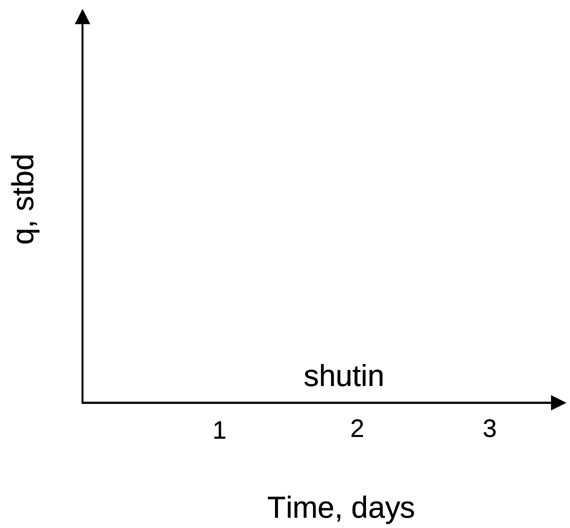
Figure 4. Example of Horner pressure buildup plot with linear discontinuity (Earlougher, 1977)

Example

Find the pressure in Well No. 2 after 3 days.







$k = 25$ md $\quad c_t = 18 \times 10^{-6}$ psia^{-1}

$\mu = 0.44$ cp $\quad B_o = 1.32$ res bbl/STB

$\phi = 16.0\%$ $\quad h = 43$ ft

$p_i = 2500$ psi