Physics 122 Test 2 study guide

This is intended to be a study guide for your second exam.

Chapter 21: Electric charges, forces, and fields; conductors and insulators.
Read all of this chapter except sections 12 and 13; you may skim section 11.

Coulomb’s law for two point charges:

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \]

Definition of electric field, \( F = qE \), and Coulomb’s law for the electric field of a
point charge

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \]

where \( \hat{r} \) points away from the charge that is causing the force.

Charge on an electron: \(-e\), and charge on a proton: \(+e\), where
\[ e = 1.6 \times 10^{-19} \text{ C}. \]

\[ k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \]

\[ \varepsilon_0 = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \]

Also, know how to break a vector into components, and how to go from
components to magnitude and direction or the reverse. (The formulas for this
are given in the sample cheat sheet, as well as in your book.)

When several forces are acting on something, we add them together to get
the total force. This idea can also be used for electric fields, since they are the
force per charge at a given location. This is often called the superposition
principle:

- \( E \) due to a bunch of charges = \( \sum (E \) due to each charge) and it also be applied to electric potentials in ch. 23.

Because of the superposition principle, we can add the fields due to a finite
number of point charges, or integrate the fields due to a continuous distribution
of charge. In all these, it is vital to remember that \( E \) is a vector, so you must split
it into its \( x \) and \( y \) (and \( z \)) components before integrating, and then integrate
each component separately. (Remember, integrating is just like adding, and you
need to split vectors into components before you add them.)

Many of the difficult charge distributions derived in this chapter are much
easier to find using Gauss’ Law, given in the next chapter, so many of the
relevant formulas for certain types of charge distributions will be listed there.

Chapter 22: Read all of it (Gauss’ Law); you may skim section 4.

Definition of lines of \( E \), and what they can point to (negative charges) and
point from (positive charges)
Definition of electric flux, and Gauss' Law

\[ \Phi_E^{\text{(closed)}} = \oiint \mathbf{E} \cdot \mathbf{n} \, dA = \frac{q_{\text{enclosed}}}{\epsilon_0} \]

For a constant \( \mathbf{E} \cdot \mathbf{n} \), \( \Phi_E = (\mathbf{E} \cdot \mathbf{n}) \times \text{(Area)} \)

(Note: many texts will write \( \mathbf{E} \cdot \mathbf{n} \, d\text{(Area)} \) as \( \mathbf{E} \cdot \mathbf{dA} \), which requires that you think of area as a vector pointing perpendicular to the plane of the area! However, that is just notation and means just what we have written here.)

Gauss' Law is always true and it is incredibly useful. However, it can be a bit intimidating at first, since it is expressed as an integral of a component of a vector field, which might be new for some of you. Some useful things to remember:

— by convention, \( \mathbf{n} \) is a unit vector on the surface being integrated which points perpendicularly outward from the volume that the surface encloses. So, \( \mathbf{E} \cdot \mathbf{n} \) measures the part of \( \mathbf{E} \) that is pointing out of the surface. (It is the ordinary dot product, which turns a vector into a number – so the value of the integral, \( \Phi_E \), is also just a number.)

— What is that number? \( \Phi_E \) represents the number of field lines going through the surface (and outward from the surface, for a closed surface). An inward field line counts as a negative outward field line. The units of \( \Phi_E \) are \( \text{N m}^2/\text{C} \), though, and not dimensionless (as a pure number of field lines would be).

— A closed surface has an inside and an outside (it could be filled with water that can’t be spilled); an open surface doesn’t. Gauss' Law applies only to closed surfaces, which are sometimes called “Gaussian Surfaces” in this context.

— In general, the full expression for \( d\text{(Area)} \) (as well as that for \( \mathbf{n} \)) varies at each point on the surface being integrated. For instance, if the surface is the top surface of a cube, then \( \mathbf{n} = \hat{z} \) and \( d\text{(Area)} = dx \, dy \). If the surface is a sphere, then \( \mathbf{n} = \hat{r} \) and \( d\text{(Area)} = r^2 \sin(\theta) \, d\theta \, d\phi \). But for simple applications where we know by symmetry that \( \mathbf{E} \cdot \mathbf{n} \) is constant on the surface, we only need to know that \( \int d\text{(Area)} = \text{the area of the surface!} \) So (as mentioned above) \( \Phi_E = (\mathbf{E} \cdot \mathbf{n}) \times \text{(Area)} \) in those cases.

— For a spherically symmetric system and a Gaussian surface that’s a sphere of radius \( r \) centered about the origin, the flux \( \Phi_E \) reduces to \( E_r \, (4\pi r^2) \). By Gauss’ Law, then:

\[ \mathbf{E} = k \frac{q_{\text{enclosed}}}{r^2 - \mathbf{r}} \]

where \( q_{\text{enclosed}} \) is the total charge inside the sphere at radius \( r \) (and \( q_{\text{enclosed}} \) is a function of \( r \), whenever there is any charge at nonzero \( r \)).

Some simple consequences of applications of Gauss' Law (all of these can also be derived by integrating the \( \mathbf{E} \) field in Coulomb’s Law, but it is much easier to use Gauss’ Law):

\( \mathbf{E} \) due to a uniform sheet of charge per area: \( \mathbf{E} = \frac{\sigma}{2\epsilon_0} \), pointing away from positive sheet or toward negative sheet
\( \mathbf{E} \) between two opposite uniform sheets of charge per area, or just outside any conductor with charge per area on the surface: \( \mathbf{E} = \frac{\sigma}{\varepsilon_0} \), pointing away from positive sheets and toward negative sheets.

\( \mathbf{E} \) a distance \( r \) away from a long line or cylinder of charge with \( \lambda = \text{charge per length} \):

\[
\mathbf{E} = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r},
\]

Inside any closed surface located within a conductor, \( Q_{\text{total}} = 0 \). (This does not apply to the outer surface of the conductor itself, necessarily; it follows from Gauss’ Law wherever \( \mathbf{E} \) must be zero, which is only truly inside the conductor.)

Another consequence: A conductor in equilibrium may only have charge on its surface(s).

Chapter 23: (Electric potential) read all of it except sections 6 and 9.

Definition of \( V \):

\[
U = qV
\]

Conservation of energy for point charges \( q \) in electrostatics:

\[
\frac{1}{2} mv^2 + qV = \text{Const.}
\]

so that \( \Delta(mv^2/2) = -q\Delta V \), giving the change in kinetic energy (or change in speed \( v \)) when the charge moves from one value of \( V \) to another.

Relation between \( \mathbf{E} \) and \( V \): \( \mathbf{E} = -\nabla V \), or backwards: \( V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{x} \).

Meaning: \( \mathbf{E} \) points in the direction of decreasing \( V \), and the magnitude of \( \mathbf{E} \) is the negative slope of \( v \) in that direction. A consequence is that lines of \( \mathbf{E} \) and surfaces of constant \( V \) are always perpendicular.

\[
\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}
\]

(For spherically symmetric situations, only the \( \hat{r} \) component of \( \nabla \) matters in the above formula.)

\( V \) of a point charge:

\[
V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} + (\text{Const.})
\]

If the choice of ground \( (V = 0) \) is as \( r \to \infty \), then the above constant is zero.

\( \Delta V = |\mathbf{E}|d \) for any uniform \( \mathbf{E} \), where the voltage drop is measured along a distance \( d \) parallel to \( \mathbf{E} \).

\( V \) due to a bunch of charges = \( \sum (V \text{ due to each charge}) \)

Units of \( V = \text{Volts}; 1 \text{V} = 1 \text{J/C} \)

Besides the Joule, another energy unit is the eV: 1 eV = (1e)(1V) = \( 1.6 \times 10^{-19} \text{J} \)