Math 586. Problem Set 4
Due May 1, 2009

1. $U(x)$ is statistically homogeneous (i.e. second-order stationary) 1-dimensional process with covariance function

$$C(h) = (1 - |h|)e^{-|h|}$$

You have observations at $x = 0.5$ and $x = 2.0$ and want to predict U at $x = 1.7$.

(a) Assume the mean is known and use simple kriging to make this prediction. Also, give the resulting kriging variance.

(b) Now suppose that the mean is NOT known; use ordinary kriging to do the estimation and to find the kriging variance.

Do not use software.

2. Let $V(\mathbf{x})$ be an IRF-0 with unknown mean and linear variogram

$$\gamma(\mathbf{h}) = 2|\mathbf{h}|$$

The observations are $V((0,1)) = 5$ and $V((2,0)) = -1$.

Set up the kriging equations (see Lecture 11, p.4), find the kriging weights, the kriging mean and variance at the prediction location $\mathbf{x}_0 = (0,0)$.
Do not use software.

3. Set up and solve the systems of kriging equations for the High Plains aquifer data (residuals from linear regression):

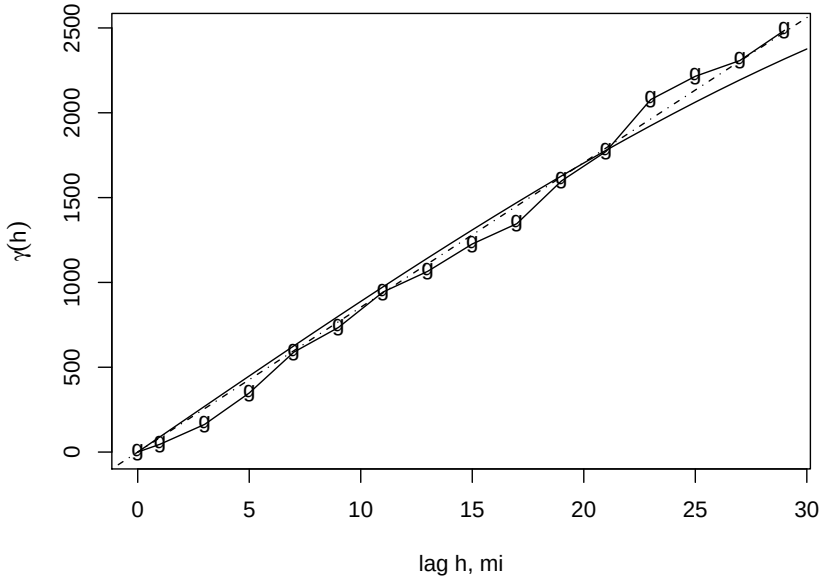
```
hipl = load('hi_plain.txt');
 x = hipl(:,6);
 y = hipl(:,7);
 V = hipl(:,9);
```

Use Ordinary Kriging with the isotropic variogram functions

(a) Linear $\gamma(h) = 85h$

(b) Spherical

$$\gamma(h) = \begin{cases} 2500\,[1.5(h/40) - 0.5(h/40)^3], & h \leq 40 \\ 2500 & h > 40 \end{cases}$$



Obtain the maps of kriging mean and standard deviation in both cases. Compare the results. (Note that the empirical variogram is unable to distinguish between these two cases.)

What is the uncertainty in (implicitly) estimating the mean?
Which of the models is more realistic for extrapolating outside the region?

4. Show that for a "pure nugget" variogram

$$\gamma(h) = \sigma^2, \quad h \neq 0$$

the ordinary kriging weights are $\lambda_j^{\text{OK}} = \dfrac{1}{n}$ and also compute the resulting kriging variance.

5. To illustrate how Equation (1) (Lecture 11, p.3) works, consider the following problem for the *Brownian sheet*:

Let $V(\mathbf{x})$ be the Brownian sheet (which is a 2-d extension of Brownian motion process) characterized by multivariate normal distribution and isotropic variogram function $\gamma(h) = h$ (here, we again use the notation $h = |\mathbf{h}|$).

(a) Let

$$\begin{aligned} Y_1 &= V(i+1, j) - V(i, j), \\ Y_2 &= V(i+1, j+1) - V(i, j) \quad \text{and} \\ Y_3 &= V(i, j+1) - V(i, j) \end{aligned}$$

(see graph). Find the covariance matrix of the vector $(Y_1, Y_2, Y_3)'$.

(b) Find the BLUE of Y_1 given Y_2 and Y_3.