Math 586. Problem Set 3
Due Feb. 25, 2009

1. This exercise is an attempt to give you some feel for correlation by examining data and estimates of correlation. For each of the following three data sets, find the associated correlation coefficients and plot the data pairs in the x-y plane.

Data Set 1			Data Set 2			Data Set 3	
X	Y		X	Y		X	Y
8	6		8	6		8	6
5	3		5	12		5	13
12	13		12	13		12	3
2	1		2	3		2	12
15	12		15	1		15	1

2. Let $Z = 2X + 3Y + 4W$ and suppose that X, Y, W have the following means and variances.

	X	Y	W
mean	10	15	12
variance	100	64	144

where X and W have correlation coefficient 0.6 while Y is independent of the other two variables. Find the mean and variance of Z.

3. X is the number of lightning strikes per minute and Y is the amount of rain (mm) observed in storms over a particular region of New Mexico. The joint probability function of X and Y was estimated and is given below. Find the best (minimum mean squared error) predictor of Y given that $X = 5$.

X / Y	3	4	5	6	7	8
5	0.05	0.03	0.01	0.01	0.02	0
6	0.04	0.03	0.01	0.01	0.02	0.01
7	0.04	0.04	0.05	0.03	0.03	0.02
8	0.03	0.05	0.05	0.04	0.05	0.03
9	0.02	0.05	0.06	0.08	0.06	0.03

4. Let $V(i,j)$, for i,j integers, be a spatially independent process (values of V at different locations are independent of each other).
For a 5-point moving average

$$m(i,j) = \frac{V(i-1,j) + V(i+1,j) + V(i,j) + V(i,j-1) + V(i,j+1)}{5}$$

find the autocovariance function of m for every vector lag $\mathbf{h}$. (A sketch could help.) For which $\mathbf{h}$ does the autocovariance equal zero?

5. Let random vector $\mathbf{Y} = (Y_1, ..., Y_4)'$ be four measurements of a MVN spatial process, with the mean vector $\boldsymbol{\mu} = (1, 0, 0, 5)'$ and covariance matrix:

$$Cov(\mathbf{Y}) = \boldsymbol{\Sigma} = \begin{bmatrix} 5 & 3 & -1 & 0 \\ 3 & 3 & 0.5 & 0.5 \\ -1 & 0.5 & 5 & -0.5 \\ 0 & 0.5 & -0.5 & 5 \end{bmatrix}$$

(a) Find the coefficients $a_1, ..., a_4$ such that $\mathbb{E}[Y_1 \mid Y_2, Y_3, Y_4] = a_1 + a_2 Y_2 + a_3 Y_3 + a_4 Y_4$. [Hint: you can either use conditioning formulas from p. 4, Lecture 5, or directly minimize MSE like on p. 1, Lecture 5.]

(b) Given values $Y_2 = 1, Y_3 = -1, Y_4 = 3$, predict the value of Y_1 and find the expected square error (MSE) of your predictor.

(c) If μ_2 is changed to 2, how would your results from part (b) change? (You might use software to invert matrix $\boldsymbol{\Sigma}$ and perform other intensive computations.)

6. Prove the conditional formulas (1) and (2) from Lecture 5, by using the formula for conditional density

$$f(x \mid y) = \frac{f(x,y)}{f_Y(y)}$$

[Hint: you'll need to find the matrix $\boldsymbol{\Sigma}^{-1}$ first.]
If the problem turns out too difficult, consider the case $\sigma_1^2 = \sigma_2^2 = 1$ instead.