Math 586. Problem Set 1
Due February 6, 2009

1. $Y = \ln K$, the natural logarithm of permeability, is assumed to be a normal random variable for a particular formation. If $\mathbb{E}[Y] = -5$, is the mean and $Var(Y) = 1.96$, is the variance, find

   (a) The probability that Y exceeds -6.

   (b) The probability that Y is between -7 and -4.

   (c) The probability that K is less than 0.005 (note: $K = e^Y$).

2. A simple model relates the percentage of petroleum recovery, R, in a given region to X, the porosity; Y, the permeability; and Z, the formation depth:

$$R = 0.4X + 900Y + 0.02Z$$

Random variable	X	Y	Z
Mean	30	0.01	75

   Find the expected value of R.

3. Which of the following are

   i. Always true

   ii. True if the random variables are independent

   iii. Generally not true

   (a) $\mathbb{E}[XYZ] = \mathbb{E}\,X\,\mathbb{E}\,Y\,\mathbb{E}\,Z$

   (b) $\mathbb{E}[X^2/Y^2] = \mathbb{E}[X^2]/\mathbb{E}[Y^2]$

   (c) $\mathbb{E}[8X + 5Y + 4W] = 8\mathbb{E}\,X + 5\mathbb{E}\,Y + 4\mathbb{E}\,W$

   (d) $Var(X + Y) = Var(X) + Var(Y)$

   (e) $Var(2X + Y) = 2Var(X) + Var(Y)$

   (f) $\mathbb{E}[e^X] = e^{\mathbb{E}\,X}$

4. One of the common confusions of students involves <u>estimates</u> versus <u>true values</u>. In the context of probability we often use probability functions assumed to be known. In practice, we may need to <u>estimate</u> these based on observations – one should <u>not</u> confuse the estimate and the true value. The following exercise is an attempt to illustrate this:

(a) Take 10 slips of paper. On five, write a 0, on the other five, a 1. Put the slips in a hat. Draw one of them. Write down the number observed. Replace the slip. Shake the slips up and draw another one. Repeat this to get a total of eight numbers. (You can also use a coin to generate 0's and 1's.)

Estimate the probability that a 1 appears by taking # of 1's observed, divided by # of trials (eight in this case).

What is the true probability of a 1?

(b) Repeat the experiment in (a) ten times. (i.e. get 10 groups of eight) and calculate an estimate of the probability each time.

Let

$$P_j = \frac{\#\text{ of ones in group } j}{8}, \qquad j = 1, 2, ..., 10$$

Calculate the estimated mean squared error,

$$\texttt{MSE} = \frac{1}{10}\sum_{j=1}^{10}(P_j - 1/2)^2$$

(c) Use the same numbers as in (b) (10 groups of eight), but now use a new estimator defined as follows:

$$\hat{r}_j = \frac{x_{j1} + 2x_{j2} + 2x_{j3} + 4x_{j4} + 4x_{j5} + 3x_{j6} + 2x_{j7} + 2x_{j8}}{20}$$

where $(x_{j1}, ..., x_{j8})$ are the eight observations in group j.
Say, your observations for Set One are 1 0 0 1 1 1 0 1. Then

$$\hat{r}_j = \frac{1 + 2\cdot 0 + 2\cdot 0 + 4\cdot 1 + 4\cdot 1 + 3\cdot 1 + 2\cdot 0 + 2\cdot 1}{20} = \frac{14}{20} = 0.70$$

Calculate the estimated mean-square error for $\hat{r}$:

$$\frac{1}{10}\sum_{j=1}^{10}(\hat{r}_j - 1/2)^2$$