

# Midterm Exam

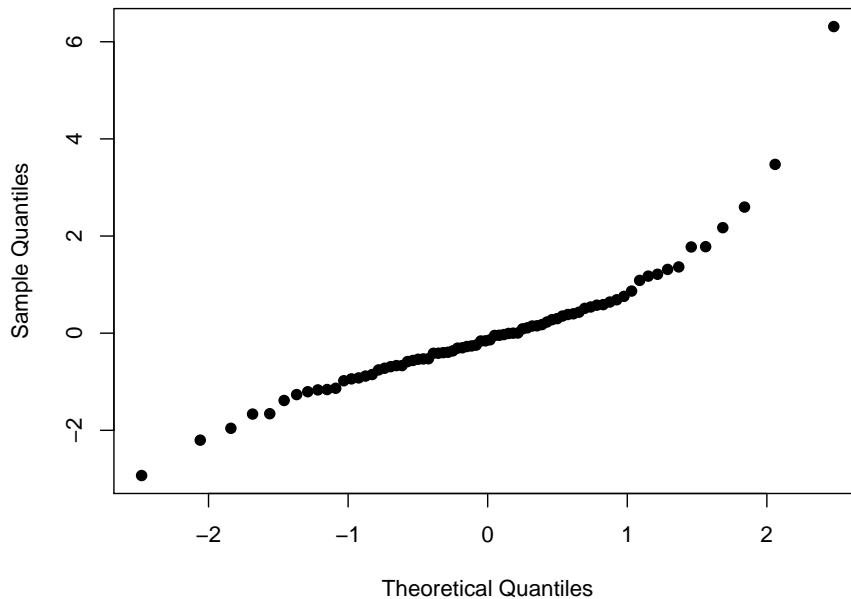
Math 586 Spring 2006

March 31, 2006

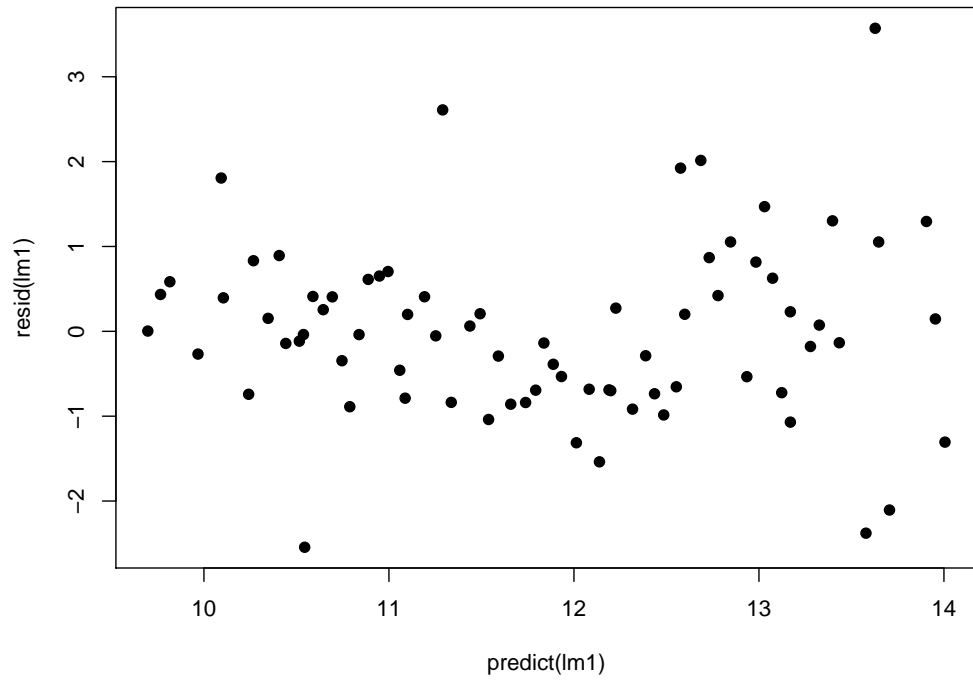
Problem	1	2	3	4	5	6	7	Total	Grade
Earned									
Possible	6	5	5	8	10	6	10	50	

1. We noticed some strange behavior when trying to estimate the variogram for West Lyons residuals. Let's try and find out some possible causes.
  - (a) Examine the normal QQ plot of the residuals. Comment. What possible complications may this cause?

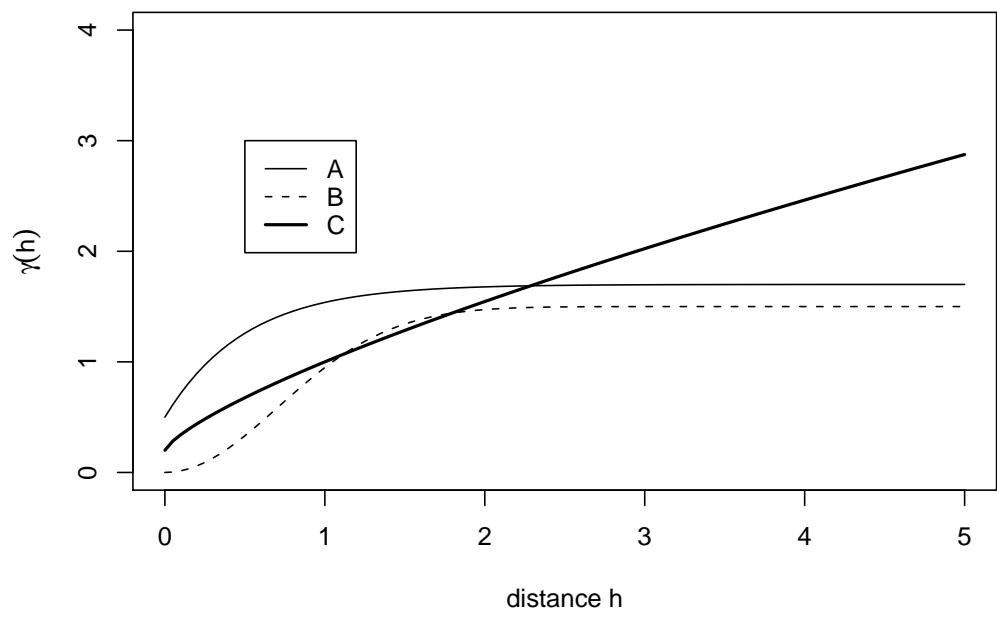
**QQ plot for West Lyons residuals**



- (b) Examine the plot of residuals *vs* predicted values for the regression on W.L. data. Do you think that the residuals can be successfully modeled as a stationary random field? What are the possible remedies?



2. For the variogram functions pictured below, estimate the nugget, sill and practical range (the range beyond which the correlation between random field values is negligible).



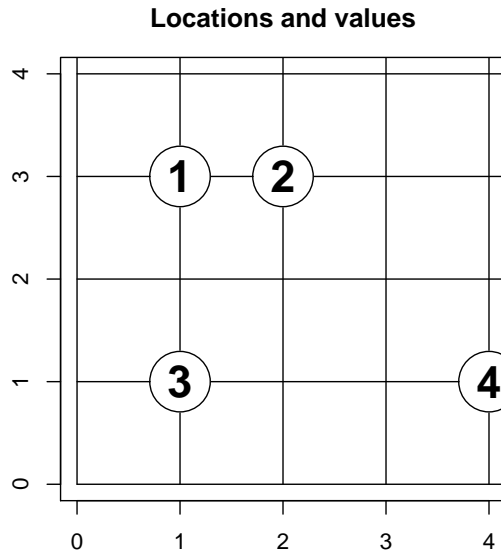
3. Let

$$W(x) = V(x) + \beta x,$$

where  $V(x)$  is “pure nugget” 1-d random field: mean 0, variance  $\sigma^2$  and  $V(x)$  is independent of  $V(y)$  when  $x \neq y$ .

Compute the variogram function for  $W$ .

4. For the data (locations and values shown) plot the variogram cloud and compute the empirical variogram for distance classes (bins)  $(0, 2.5]$  and  $(2.5, 5]$



5. The vector  $\mathbf{X} = (X_1, X_2, X_3)'$  has multivariate normal distribution with mean  $\mathbf{0}$  and covariance matrix

$$\Sigma = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(a) Compute the BLUE  $\hat{X}_1$  of  $X_1$  given  $X_2, X_3$ . That is, find the constants  $a_2, a_3$  such that

$$\hat{X}_1 = a_2 X_2 + a_3 X_3$$

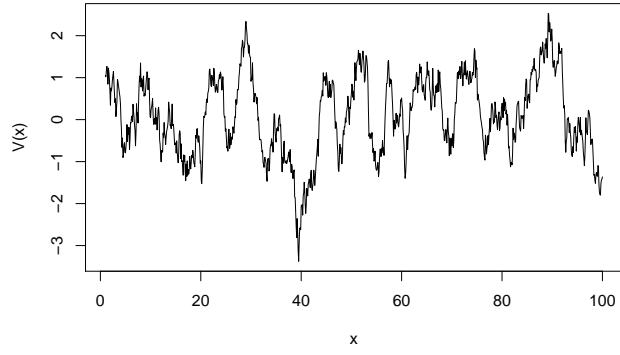
(b) How would the BLUE change if  $\mathbf{X}$  had the mean  $\boldsymbol{\mu} = (1, 2, 3)'$ ?

(c) Compute the variance of  $Y = X_1 - X_2$ .

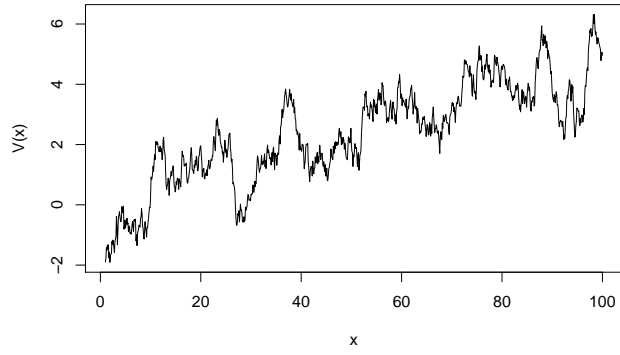
6. (a) Write the definition (maybe informal) of a second-order stationary process.

(b) Which of the following processes appear to be stationary? Explain

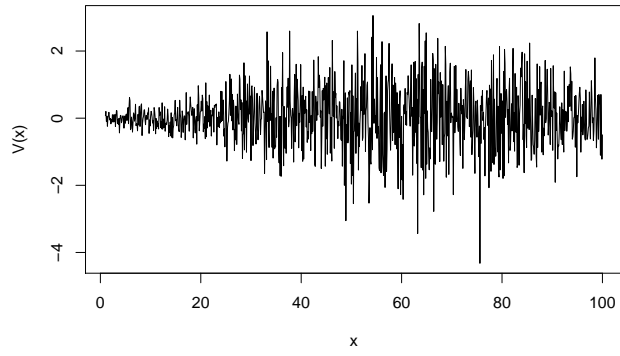
**A**



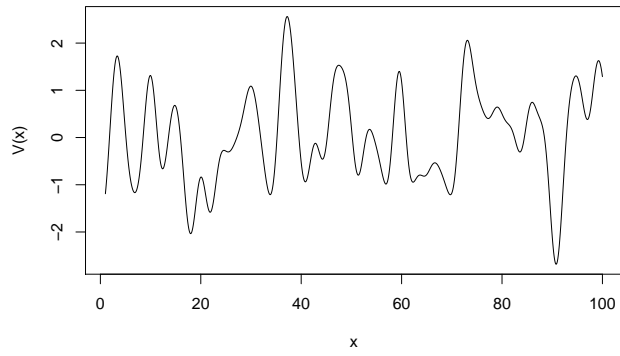
**B**



**C**



**D**



7. (a) For the linear regression problem below, estimate coefficients  $\beta$  in the equation

$$\hat{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

[Hint: matrix  $\mathbf{X}'\mathbf{X}$  will be diagonal.]

x1	x2	Y
-1	-1	3
-1	1	4
0	0	5
1	-1	7
1	1	8

- (b) Complete the ANOVA table and state whether the regression is statistically significant (critical F-value at  $\alpha = 5\%$  significance is  $F_\alpha = 19.0$ ).

	Sum of Squares	DF	Mean Square	F
Regression				
Error	0.2			--
Total	17.2		----	----

- (c) Compute  $R^2$ .