

Take-home Final

Math 483 Fall 2009

due: noon December 17 (Thursday), 2009

Note: whenever you use computer, print, write or e-mail your code and output!
Whenever the significance level α is not specified, assume $\alpha = 0.05$

Problem	1	2	3	4	5	6	Total	Grade
Earned								
Possible	10	10	10	10	10	5	55	

1. Consider the hypothesis test

$$\begin{cases} H_0 : \theta = 1 \\ H_1 : \theta \neq 1 \end{cases}$$

based on a sample X_1, \dots, X_n from the density

$$f(x; \theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

Find the distribution of Likelihood Ratio test statistic W under H_0 . [Note: since the problem is not regular, it's not $\chi^2(1)$!]

2. Are the following estimation problems regular? [Check the regularity conditions (R0), (R1), (R3), (R5); if one condition fails, you don't have to check the others.]

(a) $X_1, \dots, X_n \sim^{i.i.d.} \text{Uniform}[-\theta, \theta]$

(b) $X_1, \dots, X_n \sim^{i.i.d.} f(x; \theta)$, where

$$f(x; \theta) = \begin{cases} 2\theta^2 x^{-3}, & x \geq \theta \\ 0, & \text{otherwise} \end{cases}$$

(c) $X_1, \dots, X_n \sim^{i.i.d.} f(x; \theta)$, where

$$f(x; \theta) = \begin{cases} \theta e^{\theta(x+4)}, & x \leq -4 \\ 0, & \textit{otherwise} \end{cases}$$

3. Consider X_1, \dots, X_n a random sample from a distribution with density

$$f(x; \theta_1, \theta_2) = \begin{cases} \frac{\theta_2}{(1+x-\theta_1)^{1+\theta_2}}, & x \geq \theta_1 \\ 0, & \textit{otherwise} \end{cases}$$

(a) Find MLE's for θ_1, θ_2 . Compute the MLE's for the data set (rounded values):

112 154 119 198 112 156 123 103 115 107
125 119 128 132 106 151 103 104 116 140

(b) Compute Fisher information for θ_2 (assuming θ_1 given). Does Fisher information exist for θ_1 ?

(c) Do the data fit the distribution well?

4. Let $X_1, \dots, X_n \sim^{i.i.d.} \text{Geometric}(\theta)$, $0 < \theta < 1$.

(a) Compute Fisher information for θ .

(b) Find the MLE. Is it efficient?

(c) Conduct the Likelihood Ratio test for

$$\begin{cases} H_0 : \theta = 1/2 \\ H_1 : \theta \neq 1/2 \end{cases}$$

based on the following data:

3, 4, 1, 2, 0, 0, 2, 11, 0, 2, 1, 3, 0, 0, 2

5. Consider the regression model

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad i = 1, \dots, n$$

with $\varepsilon_i \sim^{i.i.d.} \mathcal{N}(0, \sigma^2)$, and data:

$X_i = \text{Year}$ (Year 0 = 1956),

$Y_i = \text{total soybean yield in US}$.

See the data file at http://infohost.nmt.edu/~olegm/483/Soy_reg2.txt

(a) Compute MLE's for β_0, β_1 and σ^2 .

(b) Test for the presence of a trend, that is

$$H_0 : \beta_1 = 0$$

6. Match each of the power curves below with one of the following cases:

(a) $n = 40, \alpha = 0.1$ (b) $n = 40, \alpha = 0.05$ (c) $n = 20, \alpha = 0.1$

