Math 382 Final Practice KEY

1. a) one-sample t-test
b) $H_0 : \mu = 16$ $H_A : \mu < 16$
c) skip
d) since sample mean is 16.4 hours, which is above 16, alternative hypothesis cannot be proven. Accept H_0 right away. Average time spent on chores is evidently **not** less than 16 hours.

2. a) matched pairs test
b) $H_0 : \mu_D = 0$ $H_A : \mu_D > 0$ where μ_D is average difference between garage I and garage II
c) assumptions: cost repairs do have normal distribution
d) Mean difference $\bar{x} = 240$, s = 311
T = 2.31, df = 8 p-value ≈ .025
Reject H_0 at level $\alpha = 0.05$
e) Conclusion: the garage I has, on average, higher estimates.
Can the t-test be safely used?
Maybe not, since the distribution of differences is right-skewed (outliers at 520 and 900).

3. Tobacco companies claim that an average smoker usually smokes 18.6 cigarettes a day. To test this, an independent agency took a sample of 20 smokers, on a given day they smoked average of 20.1 cigarettes, with the standard deviation of 5.3 cigarettes.
Build a **90% C.I**. for the average number of cigarettes μ smoked in a day.

t* = 1.729
$20.1 \pm 1.729*5.3/\sqrt{20}$ = [18.1, 22.1]

Looking at the above interval, does it seem credible that $\mu = 18.6$? Yes

(b) Test the hypotheses $H_0 : \mu = 18.6$ vs. $H_A : \mu \neq 18.6$ using the C.I. above.
Write your conclusion. At what level α would you test?

Since 18.6 is inside the CI, Accept H_0 : the value of 18.6 is credible.
(the test will have the level $\boldsymbol{\alpha}$ **= 100% – C** = 10%)

4. If we knew that the population st.dev. $\sigma = 3.5$, and we wanted to estimate the mean μ within the margin 0.3 with 90% confidence, what sample size would be sufficient for that?
z* = 1.645 $n = (1.645*3.5/0.3)^2 = 368$

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5. a) Xbar = 53  Ybar = 11.47   SSx = 9888, SSy = 188.02, SSxy = -1239.7,
      sx = 33.14, sy = 4.57
   b) r = SSxy/sqrt(SSx*SSy) = -0.909, it will not change
   c) y = -0.1254x + 18.115, the slope in $/km would be -0.1254*0.62 (1 km = 0.62 miles)
   d) 53.0 +- 23.7

6. a) 2000
    b) 200  (you can either use the Gamma formulas, or properties of the Sums of indep.
exponentials)
    c) use normal approx., 0.0668
    d) T = 2000 + 200(-1.645) = 1671

7.  a) 900
    b) 12.5
    c) 0.2119

8. a) yes
    b) no (success probability not constant)
    c) E(T) = 11, V(T) = 16.9

9. a) P(system works) = 1 - (1-p^2)(1-p) where p = p(t) = exp(-t/100)
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b) 116.667 hours

10. a) 2u exp(-u^2)
b) exp(-1)
c) u exp(-u)

11. a) K = 1
b) f1(x1) = x1/2 , 0 < x1 < 2, f2(x2) = 2(1-x2), 0 < x2 < 1
c) 1/(2(1-x2)), 2x2 < x1 < 2
d) 0.5
e) 8/9

12. a) 7/15
b) 14/15
c) 7/45

13. a) 1.75
b) exp(-0.25)

14. a) 0.24
b) 1/4