Math 382 Practice Final

Statistics

Part 1. Hypothesis tests

For each situation below, identify the name of the test that should be performed.

- Set up the null and alternative hypotheses.
- State the assumptions.
- Compute test statistic and p-value.
- Make your conclusion **in the words of problem** (use $\alpha = 0.05$ for both problems).

1. The administrator of a nursing home would like to do a study of staff time spent performing non-emergency type chores. In particular, she would like to test if the average time spent on non-emergency chores is less than 16 hours per day. A random sample of 25 days is taken, yielding the mean of 16.4 hours and standard deviation of 3.6 hours.

2. Insurance adjusters are concerned with the high estimates they are receiving from garage I for auto repairs compared with garage II. To verify their suspicions, each of 9 cars recently involved in an accident was taken to both garages for separate estimates of repair costs. We want to test if the average estimate is higher at garage I than at garage II. The data are given below

Car No.	1	2	3	4	5	6	7	8	9
Garage I	560	40	670	3100	640	1450	3230	520	1930
Garage II	620	50	510	2840	510	1140	2710	570	1030

Hint: calculate the differences between Garage I and Garage II estimates for each car. Then test if the mean difference equals 0.
Are there any problems with using the t-test?

Part 2. Confidence intervals

3. Tobacco companies claim that an average smoker usually smokes 18.6 cigarettes a day. To test this, an independent agency took a sample of 20 smokers, on a given day they smoked average of 20.1 cigarettes, with the standard deviation of 5.3 cigarettes.
Build a **90% C.I**. for the average number of cigarettes μ smoked in a day.

Ans. = [______________;_______________]

Using C.I. for testing:

Looking at the above interval, does it seem credible that $\mu = 18.6$? Yes No

(b) Test the hypotheses $H_0 : \mu = 18.6$ vs. $H_A : \mu \neq 18.6$ using the C.I. above.
Write your conclusion. At what level α would you test?

4. If we knew that the population st.dev. $\sigma = 3.5$, and we wanted to estimate the mean μ within the margin 0.3 with 90% confidence, what sample size would be sufficient for that?

5. For the regression analysis of the second-hand car mileage(X) and price (Y) the following results were obtained for a random sample of 10 cars:

	Mileage, 1000 mi	Price, $1000	X^2	Y^2	XY
	55	12	3025	144	660
	82	8.7	6724	75.69	713.4
	23	16	529	256	368
	15	15.5	225	240.25	232.5
	45	12.9	2025	166.41	580.5
	120	4.5	14400	20.25	540
	60	9.8	3600	96.04	588
	40	9.5	1600	90.25	380
	15	19.3	225	372.49	289.5
	75	6.5	5625	42.25	487.5
Total	530	114.7	37978	1503.63	4839.4

(a) Compute $\overline{\mathbf{X}}$, $\overline{\mathbf{Y}}$, SS_x, SS_y, s_x and s_y
(b) Compute the correlation coefficient between X and Y. Will it change if you change the miles into kilometers?
(c) Compute and interpret the intercept and slope of the regression equation
How will the slope change if you change miles into kilometers?
(d) Find a 95% confidence interval for the average price of the population of all used cars

Probability

6. The lifetime of a workstation (in days) has Gamma distribution with $\alpha = 100$ and $\beta = 20$.

a) Find the average lifetime of a workstation.
b) Find the standard deviation of lifetime.
c) Approximate the probability that the workstation will last for longer than 2300 days.
d) Find the warranty time T such that only 5% of all workstations fail before T.

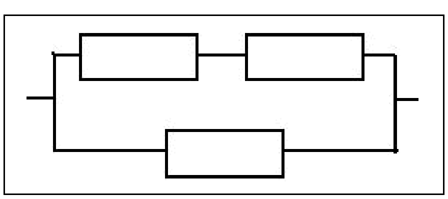
7. An electrical circuit consists of 16 resistors. Each resistance R_i is random with the mean 900 Ω and standard deviation 50 Ω. Let $\overline{X}$ = average resistance of these.

a) Find the expected value of $\overline{X}$.
b) Find standard deviation of $\overline{X}$ (assume that resistors are independent of each other).
c) Approximate the probability that $\overline{X} > 910$ Ω.

8. A test consists of 10 multiple-choice questions, with 5 variants of an answer each, and 10 true/false questions. A person is guessing answers randomly. Let X be the number of correct answers to the first 10 questions and Y the number of correct answers to the last 10 questions.

a) Are X, Y Binomial?
b) Let N = X + Y be total number of correct answers. Is it Binomial?
c) Let the student receive total of T = 3X + Y points for the test. Find E(T) and σ_T

9. For the lifetime of the following system: (the components have independent Exponential lifetimes with mean 100 hours.)



a) Find the distribution of system lifetime.
[Hint: try to evalute the CDF = P(system will last at most *t* hours)]

b) Find the expected lifetime of this system.

10. Let X_1 have exponential distribution with mean 1.
Let $U = \sqrt{X_1}$.

a) Compute the density function for U.
b) Find the probability that U > 1.

11. Let X_1 and X_2 have the joint density function given by

$$f(x_1, x_2) = \begin{cases} K, & 0 \le x_1 \le 2,\ 0 \le x_2 \le 1, \text{ and } 2x_2 \le x_1 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find the constant K
b) Find marginal densities of X_1 and X_2.
c) Find the conditional density of X_1, given $X_2 = 0.8$
d) Find $P(X_1 \le 1.5, X_2 \le 0.5)$
e) Find $P(X_2 \le 0.5 \mid X_1 \le 1.5)$

12. Out of 10 thermistors available, 2 are defective. We choose 3 thermistors randomly. Find probabilities that

a) none of the chosen are defective
b) the number of defectives does not exceed 1.
c) the first defective thermistor will be selected on the 3^{rd} try.

13. Let the number of accidents on a busy intersection have Poisson distribution, with the mean 0.25 per day.

a) Find the average number of accidents per week.
b) Find the probability that there will be no accidents on a randomly selected day.

14. The probability that an inoculated person will get flu is 20%. On the other hand, a person not inoculated will get it with probability 60%. It’s known that 90% of the population get inoculated.

a) Find the percentage of population who will get the flu.
b) Given that a person got the flu, what is the probability that he/she was not inoculated?