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KEY

Exam 1. Name (please print) ______________________

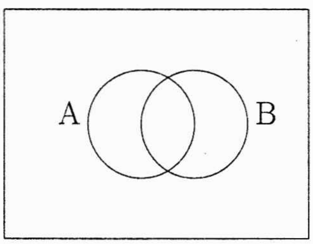
Math 382 Fall 2011. September 29, 2011.

Show work and correct notation for full credit!!! Give numerical or simple fraction answers whenever possible.

Problem	1	2	3	4	5	6	7	Total
Earned								
Possible	18	11	11	15	12	25	8	100

1. A parking facility consists of 2 lots. At a given time, there's 60% chance that the lot A is available, 50% chance that the lot B is available, and 20% chance that neither is available.

Fill in either the Venn diagram *or* a 2-way table and answer the questions:



3pt

	B	B'	
A	.3	.5−.2 =.3	.6
A'	.2	.2	1−.6=.4
	.5	1−.5 =.5	1

(a) Describe in words the meaning of the event AB'. **Find the probability of this event.**

5pt

A is available and B is not.

$P(AB') = P(B') - P(A'B') = .5 - .2 = .3$

(b) Are the events A, B independent? Explain with numbers why or why not.

5pt

need $P(AB) = P(A)P(B)$

$.3 = .6(.5)$, yes, independent

(c) If it is known that there is no parking at A, what are the chances that there is parking at B?

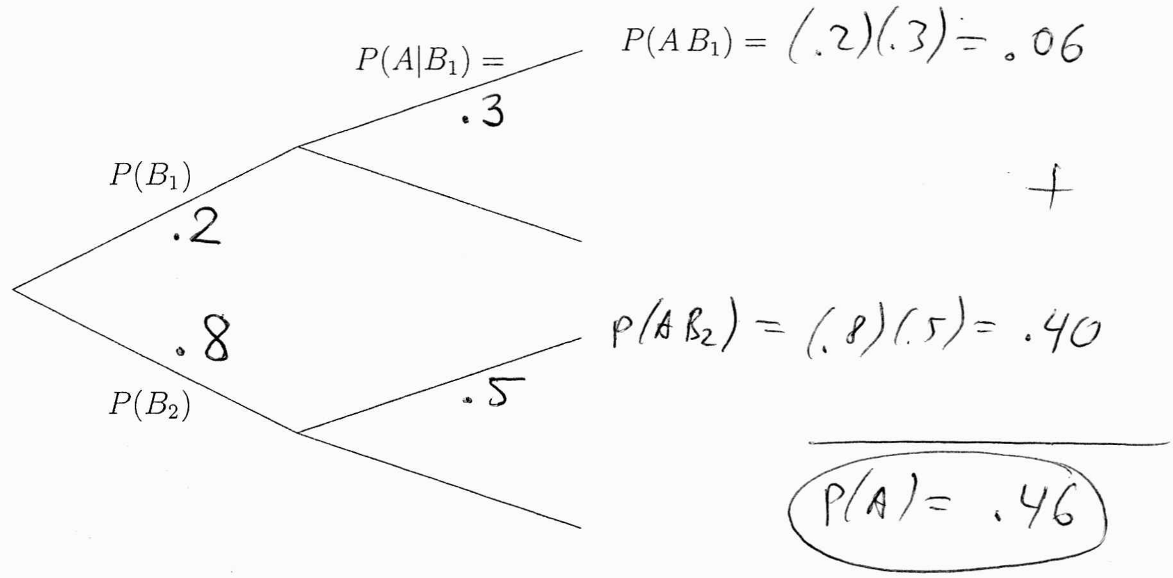
5pt

$P(B|A') = P(B)$ due to independence

$= .5$

2. At a semiconductor plant, 20% of wafers are produced by Method 1, and the rest by Method 2. Method 1 has a 30% defective rate and Method 2 (perhaps it's cheaper) has a 50% defective rate.

(a) If a wafer is randomly selected, what is the probability that it's defective?



[6]

(b) If a randomly selected wafer is defective, what is the probability that it was produced by Method 1?

[5]

$$P(B_1/A) = \frac{P(AB_1)}{P(A)} = \frac{.06}{.46} \simeq .13$$

3. The number of tropical storms per month in a region is believed to have a Poisson distribution with parameter $\lambda = 6$.

(a) Find the probability that at least 2 hurricanes will happen in a month.

[6]

$$1 - F(1) = 1 - 0.0174 = 0.9826$$

(b) Find the probability that no hurricanes will happen in a given **week**.

[5]

$$\lambda \approx \frac{6}{4} = 1.5$$

$$P(X=0) = e^{-1.5} = 0.223$$

4. The distribution of X = number of surgeries performed at a hospital in a week is given in the following table (assume $X \leq 3$).

x	0	1	2	3
$p(x)$	0.2	0.25	?	0.1

$\uparrow$

$= 1 - (0.2 + 0.25 + 0.1) = 0.45$

[2] (a) Fill in the "?"

(b) Compute the mean and standard deviation of X

[4+4] $E(X) = 0(0.2) + 1(0.25) + 2(0.45) + 3(0.1) = 1.45 = \mu$

$\mathbb{E}(X^2) = 0^2(0.2) + 1^2(0.25) + \ldots\ldots = 2.95$

$\sigma = \sqrt{V(X)} = \sqrt{\mathbb{E}(X^2) - \mu^2} = \sqrt{2.95 - 1.45^2} = 0.92$

(c) Find the value of CDF $F(2)$.

[5] $= 1 - p(3) = 0.9$

5. A space shuttle program has 10 launches planned. The probability that any launch will be delayed is 0.4.

[5]

(a) Find the probability that exactly 4 launches will be delayed

Binom(10, 0.4) $= \binom{10}{4} 0.4^4 (1-0.4)^{10-4} = 210(0.0256)(0.046656) = 0.251$

[2+3] (b) Find the expected number and the standard deviation of the number of launches delayed

$\mathbb{E}(X) = np = 4$, $\sigma = \sqrt{np(1-p)} = 1.55$

(−4) if do geometric

(c) What assumption should you make in order to answer (a),(b)?

[2] the delays are independent of each other

6. Multiple choice. Circle the best answer.

[5] each

(a) In a military exercise, the probability that a missile will destroy the target is 0.5. The probability that **at least 3** missiles are required to destroy the target is

i. 0.75 ii. 0.125 iii. 1.01 iv. 0.25

$$P(X \geq 3) = 1 - (p(1) + p(2)) = 1 - (0.5 + 0.5^2) = 0.25$$

-3 for this

(b) According to Chebyshev inequality, for a population with the mean 25 and standard deviation 5, the interval 14 to 36 is guaranteed to contain at least (pick the closest answer)

i. 50% of items ii. 80% of items iii. 20% of items iv. 90% of items

$$k = \frac{36-25}{5} = 2.2, \quad 1 - \frac{1}{(2.2)^2} \simeq .8$$

✓

(c) Out of 10 vehicles on the lot, 4 have manual transmissions. If 3 vehicles are chosen at random, the probability that **at least** one with manual transmission is chosen is

i. 50/50 ii. 4/10 iii. 5/6 iv. 2/3

$$P(Y=0) = \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \frac{6 \times 5 \times 4}{10 \times 9 \times 8} = \frac{1}{6}$$

$$P(Y \geq 1) = 1 - \frac{1}{6}$$

✓

(d) If a random variable X has the mean 5 and standard deviation 3, then the random variable $Y = 2X - 10$ will have

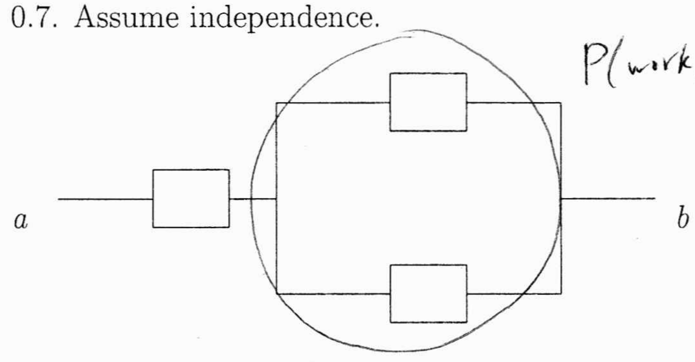
i. mean = 0 and st.dev. = 12 ii. mean = 10 and st.dev.= 6

iii. mean = 10 and st.dev. = 12 iv. mean = 0 and st.dev. = 6

✓

(e) A circuit consists of three relays, each turned on with probability 0.7. Assume independence.

$$P(\text{work}) = 1 - 0.3^2 = 0.91$$



The probability that the current flows from a to b is

i. 0.637 ii. 0.343 iii. 0.973 iv. 1.19

✓

7. True/False. No explanations are necessary, but they wouldn't hurt!

(a) Number of trials until the second success is achieved is modeled by Hypergeometric distribution.

F (Negative Bin.)

(b) A discrete random variable always has integer values.

F

(c) The cumulative distribution function (CDF) for a random variable is always non-decreasing

T

(d) Expected value of the sum (of several random variables) is the sum of expected values.

T

Extra Credit In a box with a large number of Christmas lights, there are 20% of blue lights and 20% of red. The lights are picked randomly one by one. If our goal is to get at least one blue and at least one red light, what is the expected number of lights we have to pick?

$\frac{1}{.4} + \frac{1}{.2} = 7.5$ lights