

Challenge problems for Chapters 3-6

Math 382 Fall 2011

Note: Even if you solve one or two problems, you are encouraged to turn them in.

1. Jack is tossing a coin until he records the combination “HT” (H is for Heads and T is for Tails). Jill is tossing a coin until she records the combination “HH”.
 - (a) Find the expected number of tosses for Jack.
 - (b) Find the expected number of tosses for Jill.
2. We are collecting figurines that come with a purchase of a box of teabags. There are 10 different figurines, each of them equally likely to show up in each new box. How many boxes do we have to purchase, on average, to collect all 10?
3. Two round chips, with radius 1 each, are randomly thrown on a round table with radius 3. What is the probability that the chips overlap?
4. Three points are randomly located around the unit circle. [That is, the set of (x, y) such that $x^2 + y^2 = 1$.] Find the probability that there exists a semi-circle containing all three points.
5. Random variables are repeatedly drawn from the same continuous distribution, with the density function f . (Actually, the exact shape of f is irrelevant.) Let's call it a *record* when the current observation is higher than all previous ones. For example, if a sequence of 3.65, 2.7, 4.11, 3.44, 5.90 is observed, then 3.65, 4.11, and 5.90 are the records. Find the expected number of records when the total of $n = 100$ such variables are drawn. **[Hint:** You can solve this problem step by step starting from $n = 1$.]
6. Jack and Jill are making random rectangles. They draw the side measurements from some positive random variable X . Jack is lazy, he draws just once, and makes a square. Jill draws a pair of independent random variables with the same distribution, X_1 and X_2 . Whose rectangle has a larger area? [Explain with the proof.]

7. Jack and Jill are proofreading. After reading a text, Jack has found 15 errors. Jill read the text independently, and found 20 errors, among them 10 of the same errors that Jack found. Assuming that all errors are equally easy to find, estimate how many errors are still left undetected in the text.
8. Suppose that X is a positive-valued continuous random variable, with some density f and CDF F . You choose the value of c , and then the random variable X is drawn. You get c dollars if $X > c$ and nothing if $X \leq c$.
- (a) Set up the equations to determine the best choice of c that maximizes your expected winnings.
- (b) Find the optimal c in case when X is an exponential random variable.
9. *Cauchy* random variable has the density function

$$f(x) = \frac{1}{\pi(1+x^2)}$$

If we take a pair of independent random variables with the same Cauchy distribution, X_1 and X_2 , then it turns out that $X_1 + X_2$ has the same distribution as $2X$.

- (a) Prove the above statement.
- (b) Argue from the above statement, that variance of X has to be infinite.