**Math 283 – Summer 2011** **Study Sheet for Exam #2**

Solutions

**1.** a) Two-sample t-test

b) $H_0 : \mu_1 = \mu_2$ $H_A : \mu_1 \neq \mu_2$

c) Assumptions: drug potency has normal distribution. Samples are random.

d) Test statistic $t = \dfrac{85.2 - 76.0}{\sqrt{10.3^2/10 + 12.5^2/10}} = 1.80$ df = 9 p-value = 0.105, is above 0.05

e) Conclusion: there is no significant difference in potency between current and after-storage drug.

**2.** a) one-sample t-test

b) $H_0 : \mu = 16$ $H_A : \mu < 16$

c) skip

d) since sample mean is 16.4 hours, which is above 16, alternative hypothesis cannot be proven. Accept $H_0$ right away. Average time spent on chores is evidently **not** less than 16 hours.

**3.** a) matched pairs test

b) $H_0 : \mu_D = 0$ $H_A : \mu_D > 0$ where $\mu_D$ is average difference between garage I and garage II

c) assumptions: cost repairs do have normal distribution

d) Mean difference $\bar{x} = 240$, s = 311

T = 2.31, df = 8 p-value ≈ .025

Reject $H_0$ at level $\alpha = 0.05$

e) Conclusion: the garage I has, on average, higher estimates.

Can the t-test be safely used?

Maybe not, since the distribution of differences is right-skewed (outliers at 520 and 900), and *n* is small

**4.** a) z-test for proportion

b) $H_0 : p = 0.90$ $H_A : p \neq 0.90$

c) assumption: $np_0$ and $n(1-p_0)$ both above 10. Checks out.

d) p-hat = 2678/3065 = 0.874 z = – 4.85 p-value = .000, very small

e) Conclusion: the proportion graduating that high school is significantly different from the district average. (In fact, it's lower)

**5.** Tobacco companies claim that an average smoker usually smokes 18.6 cigarettes a day. To test this, an independent agency took a sample of 20 smokers, on a given day they smoked average of 20.1 cigarettes, with the standard deviation of 5.3 cigarettes.

Build a **90% C.I**. for the average number of cigarettes $\mu$ smoked in a day.

t* = 1.729

$20.1 \pm 1.729 * 5.3/\sqrt{20}$ = [18.1, 22.1]

Looking at the above interval, does it seem credible that $\mu = 18.6$? <u>Yes</u>

(b) Test the hypotheses $H_0 : \mu = 18.6$ vs. $H_a : \mu \neq 18.6$ using the C.I. above.

Write your conclusion. At what level $\alpha$ would you test?

Since 18.6 is inside the CI, Accept $H_0$ : the value of 18.6 is credible.

(the test will have the level $\boldsymbol{\alpha}$ **= 100% – C** = 10%)

**6.** If we knew that the population st.dev. $\sigma = 3.5$, and we wanted to estimate the mean $\mu$ within the margin 0.3 with 90% confidence, what sample size would be sufficient for that?

$z^* = 1.645$ $n = (1.645 * 3.5/0.3)^2 = 368$

**7.** To determine the occurrence of side effects for a new drug, a study is planned.

(a) How large should a sample size be to determine the proportion of patients with minor side effects to within ±2% with the confidence 90%?

$n = (1.645/0.02)^2\ 0.25 = 1691$

(b) If in the sample of n = 100 patients 18 had minor side effects, find a 98% confidence interval for the proportion of all likely patients who would have minor side effects.

$\hat{p} = 0.18$ $0.18 \pm 2.326\sqrt{0.18(1-0.18)/100} = 0.180 \pm 0.089 =$ [0.091; 0.269]

(c) $n\hat{p} = 18$ and $n(1-\hat{p}) = 82$ should be both above 10 : true. Normal Approximation is valid.

As the confidence level increases, the Confidence interval (circle one)

Gets wider

As the sample size increases, the Confidence interval (circle one)

Gets narrower

**5.31** a) $p = 0.1137$

b) using $\mu = np = 136.4$

c) using $\mu = 136.4$ and $\sigma = 11.0$, obtain $z = (100 - 136.4)/11.0 = -3.31$ and $P(Z < -3.31) = 0.0005$

since $np = 1200(0.1137) > 10$, the Normal approx. is good.

**5.75** Control limits are $\mu \pm 3\sigma/\sqrt{5} =$ 4.0689 to 4.4311

You might also be required to find something like

$P(\overline{X} > 4.2) = P[Z > (4.2 - 4.25)/(0.135/\sqrt{5})] = P(Z > -0.33) = 1 - TA(-0.33) = 0.6293$

* Chi-square test: Review Homework 23