Sample problems for the Final Exam

Part I: Chapter 16.

1. Compute the curl and divergence of the vector field $\mathbf{F} =< x + \sin yz, y + \sin xz, z + \sin xy >$.

2. (a) Find the unit normal $\mathbf{n}$ to the surface $z = 4 - x^2 - y^2$.
   (b) Find the area and centroid of the part of the surface that lies above $z = 0$, with density $\delta \equiv 1$.

3. Find the mass of the wire with line density $\delta(x, y, z) = \sqrt{z + 9/2}$, following the curve C: $x = 2\sqrt{2}t$, $y = 2 - t$, $z = t^2/2$, $0 \le t \le 1$.

4. Evaluate the integral of $f(x, y) = xy^2$ along the curve C consisting of two line segments connecting points $(-1, 2)$, $(-1, 5)$ and $(3, 5)$, in that order.

5. Find the potential function for the vector field $< 2x + y, x + 2y, 2z >$.

6. Check for independence of path and evaluate the line integral
$$\int_{(0,0)}^{(1,-1)} 2xe^y\, dx + x^2 e^y\, dy$$

7. Use the Green's theorem to evaluate
$$\int_C (\arctan x - y)dx + (e^y - x)dy,$$
   where C is the part of the ellipse $(x/5)^2 + (y/2)^2 = 1$ where $x \le 5\sqrt{3}/2$, traversed counterclockwise.

8. Let $\mathbf{F} = x\ \mathbf{i} + y\ \mathbf{j}$, and S be the hemisphere $z = \sqrt{9 - x^2 - y^2}$. Find the flux of the vector field $\mathbf{F}$ through the surface by direct computation.

9. Using Divergence Theorem, evaluate the flux from previous problem.

Answers:

1. $\nabla \times \mathbf{F} =< x\cos xy - x\cos xz,\ y\cos yz - y\cos xy,\ z\cos xz - z\cos yz >$,
   $\nabla \cdot \mathbf{F} = 3$.

2. (a) $\mathbf{n} = \dfrac{< 2x, 2y, 1 >}{\sqrt{1+4x^2+4y^2}}$ (b) $A = (\pi/6)(17^{3/2} - 1)$, $\overline{x} = \overline{y} = 0$,
   $\overline{z} = 17/4 - (3/20)(17^{5/2} - 1)/(17^{3/2} - 1) \approx 1.665$

3. $(9 + 1/3)/\sqrt{2}$ 4. 61

5. $f(x, y, z) = x^2 + xy + y^2 + z^2$. 6. $1/e$

7. $5\sqrt{3} + 1/e - e$ 8-9. 36π

NOTE: *The answers have been carefully checked, however, errors are still possible!*