

### Sample problems for Exam 3.

1. Sketch the region of integration and evaluate the integral

$$\int_0^2 \int_{y/2}^1 y e^{x^3} dx dy$$

2. Set up the integral for the volume  $V$  of the solid  $T$  bounded by the planes  $z = 6$  and  $z = 2y$  and by the parabolic cylinders  $y = x^2$  and  $y = 2 - x^2$ . (Set up, do not evaluate.)
3. Find the area of the region  $R$  bounded on the inside by the circle  $r = 1$  and on the outside by  $r = 2 + \cos \theta$ .
4. Find the volume bounded by paraboloids  $z = x^2 + y^2$  and  $z = 4 - 3x^2 - 3y^2$ .
5. Set up the integral for the mass  $m$  of the pyramid  $T$  whose vertices are  $O(0, 0, 0)$ ,  $P(2, 0, 0)$ ,  $Q(0, 3, 0)$  and  $R(0, 0, 6)$ , if its density is given by  $\delta(x, y, z) = z$ . (Set up, do not evaluate.)
6. Let  $T$  be the first-octant (i.e. for  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ ) portion of the solid ball with constant density  $\delta \equiv 1$  bounded by the sphere  $x^2 + y^2 + z^2 = 4$ . Find  $z$ -component of the centroid of  $T$ . (Hint: you may use the fact that the volume of the sphere with radius  $a$  is  $4\pi a^3/3$ .)
7. Find the surface area of the part of the cylinder  $x^2 + z^2 = 2$  that lies within the cylinder  $x^2 + y^2 = 2$ .
8. Sketch the region of integration and evaluate the integral by using polar coordinates:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2} dx dy$$

9. Solve the initial value problem

$$\frac{dy}{dx} = y^2 + 1, \quad y(1) = 0$$

10. Solve the initial value problem

$$y'' + 2y' + 5 = 0, \quad y(0) = 1, y'(0) = 1$$

**Answers:**

1.  $\frac{2}{3}(e - 1)$

2.  $\int_{-1}^1 \int_{x^2}^{2-x^2} (6 - 2y) dy dx$

3.  $\frac{7}{2}\pi$

4.  $2\pi$

5.  $m = \int_0^2 \int_0^{\frac{6-3x}{2}} \int_0^{6-3x-2y} z dz dy dx$

6.  $3/4$

7.  $16$

8.  $\frac{\pi}{4} \ln 2$

9.  $\tan(x - 1)$

10.  $e^{-t}(\cos 2t + \sin 2t)$

**NOTE:** *The answers have been carefully checked, however, errors are still possible!*