Sample problems for Exam 3.

1. Sketch the region of integration and evaluate the integral

\[ \int_0^2 \int_{y/2}^1 ye^{x^3} \, dx \, dy \]

2. Set up the integral for the volume \( V \) of the solid \( T \) bounded by the planes \( z = 6 \) and \( z = 2y \) and by the parabolic cylinders \( y = x^2 \) and \( y = 2 - x^2 \). (Set up, do not evaluate.)

3. Find the area of the region \( R \) bounded on the inside by the circle \( r = 1 \) and on the outside by \( r = 2 + \cos \theta \).

4. Find the volume bounded by paraboloids \( z = x^2 + y^2 \) and \( z = 4 - 3x^2 - 3y^2 \).

5. Set up the integral for the mass \( m \) of the pyramid \( T \) whose vertices are \( O(0, 0, 0), P(2, 0, 0), Q(0, 3, 0) \) and \( R(0, 0, 6) \), if its density is given by \( \delta(x, y, z) = z \). (Set up, do not evaluate.)

6. Let \( T \) be the first-octant (i.e. for \( x \geq 0, y \geq 0, z \geq 0 \)) portion of the solid ball with constant density \( \delta \equiv 1 \) bounded by the sphere \( x^2 + y^2 + z^2 = 4 \). Find \( z \)-component of the centroid of \( T \). (Hint: you may use the fact that the volume of the sphere with radius \( a \) is \( 4\pi a^3 / 3 \).)

7. Find the surface area of the part of the cylinder \( x^2 + z^2 = 2 \) that lies within the cylinder \( x^2 + y^2 = 2 \).

8. Sketch the region of integration and evaluate the integral by using polar coordinates:

\[ \int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1 + x^2 + y^2} \, dx \, dy \]

9. Solve the initial value problem

\[ \frac{dy}{dx} = y^2 + 1, \quad y(1) = 0 \]

10. Solve the initial value problem

\[ y'' + 2y' + 5 = 0, \quad y(0) = 1, y'(0) = 1 \]
Answers:

1. $\frac{2}{3}(e - 1)$

2. $\int_{-1}^{1} \int_{x^2}^{2-x^2} (6 - 2y) \, dy \, dx$

3. $\frac{7}{2}\pi$

4. $2\pi$

5. $m = \int_{0}^{2} \int_{0}^{\frac{6-3z}{2}} \int_{0}^{6-3x-2y} z \, dz \, dy \, dx$

6. $3/4$

7. $16$

8. $\frac{\pi}{4}\ln 2$

9. $\tan(x - 1)$

10. $e^{-t}(\cos 2t + \sin 2t)$

**NOTE:** The answers have been carefully checked, however, errors are still possible!