Sample problems for Exam 2.

1. State and sketch the largest possible domain of definition of the function
   \[ f(x, y) = \arcsin(x^2 + y^2 - 1) \]

2. Write an equation of the plane tangent to the paraboloid \( z = 5 - 2x^2 - y^2 \) at the point \( P(1, 1, 2) \).

3. For the plane \( 12x + 4y + 3z = 169 \), find the point \( P(x, y, z) \) on the plane closest to the point \( Q(0, 0, 0) \).

4. Approximate \( \sqrt{2(2.02)^3 + (2.97)^2} \)
   by using the differential (linear approximation).

5. Given
   \[ w = \ln(x^2 + y^2 + z^2), \quad x = s - t, \quad y = s + t, \quad z = 2\sqrt{st}, \]
   find \( \frac{\partial w}{\partial s} \) and \( \frac{\partial w}{\partial t} \).

6. Find the directional derivative of \( f(x, y, z) = \sqrt{xyz} \) at the point \( P(2, -1, -2) \) in the direction of \( \mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} \).

7. Given \( f(x, y) = 2x^2 + 3xy + 4y^2 \), find the maximum directional derivative of \( f \) at the point \( P(1, 1) \) and the direction in which it occurs.

8. The plane \( x + y + z = 12 \) intersects the paraboloid \( z = x^2 + y^2 \) in an ellipse. Find the highest and lowest points of the ellipse.

9. A rectangular box of volume 1500 in\(^3\) with 5 sides (no top) is made; the bottom costs $3 per square inch, and the four other sides cost $1 per square inch. Find the measurements \( x, y, z \) that provide the smallest cost.
Answers:

1. \( x^2 + y^2 \leq 2 \)
2. \( 4x + 2y + z = 8 \)
3. \((12, 4, 3)\)
4. 5.03
5. \( \frac{\partial w}{\partial s} = \frac{2}{s + t}, \quad \frac{\partial w}{\partial t} = \frac{2}{s + t} \)
6. 1/2
7. \( \sqrt{170}, \) the same direction as \( <7, 11> \).
8. lowest \((2, 2, 8)\), highest \((-3, -3, 18)\)
9. \( x = y = 10, \ z = 15. \)

**NOTE:** The answers have been carefully checked, however, errors are still possible!