

Sample problems for Exam 2.

1. State and sketch the largest possible domain of definition of the function

$$f(x, y) = \arcsin(x^2 + y^2 - 1)$$

2. Write an equation of the plane tangent to the paraboloid $z = 5 - 2x^2 - y^2$ at the point $P(1, 1, 2)$.
3. For the plane $12x + 4y + 3z = 169$, find the point $P(x, y, z)$ on the plane closest to the point $Q(0, 0, 0)$.

4. Approximate

$$\sqrt{2(2.02)^3 + (2.97)^2}$$

by using the differential (linear approximation).

5. Given

$$w = \ln(x^2 + y^2 + z^2), \quad x = s - t, \quad y = s + t, \quad z = 2\sqrt{st},$$

find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$.

6. Find the directional derivative of $f(x, y, z) = \sqrt{xyz}$ at the point $P(2, -1, -2)$ in the direction of $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$.
7. Given $f(x, y) = 2x^2 + 3xy + 4y^2$, find the maximum directional derivative of f at the point $P(1, 1)$ and the direction in which it occurs.
8. The plane $x + y + z = 12$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the highest and lowest points of the ellipse.
9. A rectangular box of volume 1500 in^3 with 5 sides (no top) is made; the bottom costs \$3 per square inch, and the four other sides cost \$1 per square inch. Find the measurements x, y, z that provide the smallest cost.

Answers:

1. $x^2 + y^2 \leq 2$
2. $4x + 2y + z = 8$
3. $(12, 4, 3)$
4. 5.03
5. $\frac{\partial w}{\partial s} = \frac{2}{s+t}, \quad \frac{\partial w}{\partial t} = \frac{2}{s+t}$
6. $1/2$
7. $\sqrt{170}$, the same direction as $\langle 7, 11 \rangle$.
8. lowest $(2, 2, 8)$, highest $(-3, -3, 18)$
9. $x = y = 10, z = 15$.

NOTE: *The answers have been carefully checked, however, errors are still possible!*