Sample problems for Exam 1.

1. Find (*approximate to the nearest degree*) the three angles of the triangle with the vertices $A(1,1,1)$, $B(3,-2,3)$ and $C(3,4,6)$.

2. Find (*exactly*) the area of the triangle with the vertices $A(1,3,-2)$, $B(2,4,5)$ and $C(-3,-2,2)$.

3. Find the volume of the pyramid with vertices $O(0,0,0)$, $A(3,2,-1)$, $B(-2,5,1)$ and $C(2,1,5)$.

4. Write symmetric equation for the straight line passing through $P(2,-3,4)$ and perpendicular to the plane with the equation $2x - y + 3z = 4$.

5. Find an equation of the plane through the three points $P(2,4,-3)$, $Q(3,7,-1)$ and $R(4,3,0)$.

6. Determine whether the line $\mathbf{r}(t) = (3+2t)\mathbf{i} + (6-5t)\mathbf{j} + (2+3t)\mathbf{k}$ and the plane $3x + 2y - 4z = 1$ intersect or are parallel. If they intersect, find the point P of intersection.

7. The position vector of a moving particle is $\mathbf{r}(t) = 12t\ \mathbf{i} + 5\sin(2t)\ \mathbf{j} - 5\cos(2t)\ \mathbf{k}$. Find its velocity $\mathbf{v}$, speed v and acceleration $\mathbf{a}$ at time t.

8. Find the curvature κ of the space curve with position vector $\mathbf{r}(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + t^2\ \mathbf{k}$.

9. For a particle on the plane, moving from point $\mathbf{r}_0 = (0,0)$ with the initial velocity $\mathbf{v}_0 = \mathbf{i} + \mathbf{j}$ and acceleration $\mathbf{a}(t) = (4-t)\ \mathbf{i}$, find its velocity and position vector at time $t > 0$.

10. Find the arc length of the space curve $\mathbf{r}(t) = t\ \mathbf{i} + t^2\ \mathbf{j} + (2t^3/3)\ \mathbf{k}$ from $t = 0$ to $t = 3$.

11. Describe 3-dimensional surfaces, find their traces and sketch:
    a)
    $$2z^2 = x^2 + y^2 + 4$$
    b)
    $$r = \cos\theta$$

Answers:

1. $A \approx 79^o, B \approx 64^o, C \approx 37^o$
2. $\sqrt{2546}/2$
3. 18
4. $\dfrac{x-2}{2} = -(y+3) = \dfrac{z-4}{3}$
5. $11x + y - 7z = 47$
6. $P(9/2, 9/4, 17/4)$
7. $\mathbf{v} = 12\ \mathbf{i} + 10\cos(2t)\ \mathbf{j} + 10\sin(2t)\ \mathbf{k}$, $v = 2\sqrt{61}$,
   $\mathbf{a} = -20\sin(2t)\ \mathbf{j} + 20\cos(2t)\ \mathbf{k}$
8. $\kappa = \dfrac{\sqrt{5+4t^2}}{(1+4t^2)^{3/2}}$
9. $\mathbf{v}(t) = (-t^2/2 + 4t + 1)\ \mathbf{i} + \ \mathbf{j}$, $\mathbf{r}(t) = (-t^3/6 + 2t^2 + t)\ \mathbf{i} + t\ \mathbf{j}$
10. $v(t) = 2t^2 + 1$, length $= 21$.
11. a) Hyperboloid of two sheets (obtained by revolving the hyperbola $2z^2 = x^2 + 4$ in xz-plane around z-axis.)
    b) In polar coordinates (r, θ) it's a circle with center $(x = 1/2, y = 0)$ and radius $1/2$; adding z-coordinate makes it a circular cylinder.

NOTE: *The answers have been carefully checked, however, errors are still possible!*