1. Let $x = 2\sin\theta$, $dx = 2\cos\theta d\theta$; then

$$\int \frac{x^2}{\left(4-x^2\right)^{3/2}}dx = \int \frac{4\sin^2\theta}{\left(4-4\sin^2\theta\right)^{3/2}}2\cos\theta d\theta = \int \tan^2\theta d\theta$$

$$= \tan\theta - \theta + C = \frac{x}{\sqrt{4-x^2}} - \arcsin\left(\frac{x}{2}\right) + C$$

2. Let $u = \sin x,\ du = \cos x dx$

$$\int \cos^3 x \sin^2 x dx = \int \cos^2 x u^2 du = \int \left(1-u^2\right)u^2 du$$

$$= \frac{u^3}{3} - \frac{u^5}{3} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

3. Let $u = x,\ du = dx$ and $dv = \sec x \tan x dx,\ v = \sec x$ so

$$\int x\sec x \tan x dx = x\sec x - \int \sec x dx = x\sec x - \ln\left|\sec x + \tan x\right| + C$$

4. $\dfrac{x^2+8x-3}{x\left(x^2+3\right)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+3}$, solving for *A, B, C* we get $A=-1,\ B=2,\ C=8$ so

$$\int \frac{x^2+8x+3}{x^3+3x}dx = \int\left(-\frac{1}{x} + \frac{2x}{x^2+3} + \frac{8}{x^2+3}\right)dx = -\ln|x| + \ln\left(x^2+3\right) + \frac{8\sqrt{3}}{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

5. Let $x = \tan\theta,\ dx = \sec^2\theta d\theta$,

$$\int \frac{dx}{x^2\sqrt{x^2+1}} = \int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\tan^2\theta+1}}d\theta = \int \frac{\cos\theta}{\sin^2\theta}d\theta$$

$$= -\frac{1}{\sin\theta} + C = -\frac{\sqrt{1+x^2}}{x} + C$$

6. Let $u = \tan x,\ du = \sec^2 x dx$, $\displaystyle\int \tan^4 x \sec^2 x dx = \int u^4 du = \frac{u^5}{5} + C = \frac{\tan^5 x}{5} + C$

7. $\dfrac{x^3+4x^2}{x^2+4x+3} = x + \dfrac{-3x}{(x+3)(x+1)} = x + \dfrac{3}{2}\dfrac{1}{x+1} - \dfrac{9}{2}\dfrac{1}{x+3}$,

$$\int \frac{x^3+4x^2}{x^2+4x+3}dx = \int\left(x + \frac{3}{2}\frac{1}{x+1} - \frac{9}{2}\frac{1}{x+3}\right)dx = \frac{x^2}{2} + \frac{3}{2}\ln|x+1| - \frac{9}{2}\ln|x+3| + C$$

8. Let $u = e^{-2x},\ du = -2e^{-2x}dx$ and $dv = \sin(3x)dx,\ v = -\frac{1}{3}\cos(3x)$

$$\int e^{-2x}\sin(3x)dx = I = -\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{3}\int e^{-2x}\cos(3x)dx,$$

now let $u = e^{-2x},\ du = -2e^{-2x}dx$ and $dv = \cos(3x)dx,\ v = \frac{1}{3}\sin(3x)$ so

$I = -\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{3}\int e^{-2x}\cos(3x)dx = -\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{9}e^{-2x}\sin 3x - \frac{4}{9}I$ thus

$$I = -\frac{3}{13}e^{-2x}\cos(3x) - \frac{2}{13}e^{-2x}\sin(3x) + C$$

9. $S = \int_1^e 2\pi x\sqrt{1+\frac{1}{x^2}}dx = \int_1^e 2\pi\sqrt{1+x^2}dx$, now let $x = \tan\theta,\ dx = \sec^2\theta d\theta$ so

$S = \int_1^e 2\pi x\sqrt{1+\frac{1}{x^2}}dx = \int_1^e 2\pi\sqrt{1+x^2}dx = \int_*^* \sec^3\theta d\theta$

$= \pi x\sqrt{1+x^2}\Big|_1^e + \pi\ln\left|\sqrt{1+x^2}+x\right|\Big|_1^e = \pi\left[e\sqrt{1+e^2} - \sqrt{2}\right] + \pi\left[\ln\left(\sqrt{1+e^2}+e\right) - \ln\left|\sqrt{2}+1\right|\right]$

10. $V = \int_0^1 2\pi x e^x dx$ now let $u = 2\pi x,\ du = 2\pi dx$ and $dv = e^x dx,\ v = e^x$

so $V = \int_0^1 2\pi x e^x dx = 2\pi x e^x\Big|_0^1 - 2\pi\int_0^1 e^x dx = 2\pi x e^x\Big|_0^1 - 2\pi e^x\Big|_0^1 = 2\pi$

11. $V = \int_1^e \left(\pi(1)^2 - \pi(\ln x)^2\right)dx = \int_1^e \pi dx - \int_1^e \pi(\ln x)^2 dx = \pi$ (use integration by parts)

12. $dy = \frac{\sqrt{x^2-4}}{x}dx \Rightarrow \int dy = \int \frac{\sqrt{x^2-4}}{x}dx$, let $x = 2\sec\theta,\ dx = 2\sec\theta\tan\theta d\theta$, so

$y = \int \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta}2\sec\theta\tan\theta d\theta = \int 2\tan^2\theta d\theta = 2\tan\theta - 2\theta + C$

$= \sqrt{x^2-4} - 2\,\text{arcsec}\left(\frac{x}{2}\right) + C$

Now $y(2) = 0 \Rightarrow 0 = \sqrt{4-4} - 2\,\text{arcsec}(1) + C \Rightarrow C = 0$, so $y = \sqrt{x^2-4} - 2\,\text{arcsec}\left(\frac{x}{2}\right)$.

13. $dy = \frac{2\sqrt{3}}{3x^4 + 4x^2 + 1} dx \Rightarrow \int dy = \int \frac{2\sqrt{3}}{3x^4 + 4x^2 + 1} dx$, now

$\frac{2\sqrt{3}}{3x^4 + 4x^2 + 1} = \frac{2\sqrt{3}}{(3x^2 + 1)(x^2 + 1)} = \frac{Ax + B}{3x^2 + 1} + \frac{Cx + D}{x^2 + 1}$, solving

$A = C = 0$ and $D = -\sqrt{3}$ and $B = 3\sqrt{3}$, so

$\int dy = \int \frac{2\sqrt{3}}{3x^4 + 4x^2 + 1} dx \Rightarrow y = \int \left(\frac{3\sqrt{3}}{3x^2 + 1} - \frac{\sqrt{3}}{x^2 + 1} \right) dx = 3\arctan\left(\sqrt{3}x\right) - \sqrt{3}\arctan x + C$

$y(1) = -\frac{\pi\sqrt{3}}{4}$, so $C = -\pi$. Thus $y = 3\arctan\left(\sqrt{3}x\right) - \sqrt{3}\arctan x - \pi$