

1. A cyclist rides down a long straight road at a velocity (in m/min) given by $v(t) = 400 - 20t$ for $0 \leq t \leq 10$ min.

a. How far does the cyclist travel in the first 10 min?

$$\int_0^{10} (400 - 20t) dt = 3000 \text{ m}$$

b. How far has the cyclist traveled when her velocity is 250 m/min?

When is $v(t) = 400 - 20t = 250$ at $t = \frac{15}{2}$, so the cyclist traveled $\int_0^{15/2} (400 - 20t) dt = \frac{4875}{2}$ m

2. Find the area of the region bounded by $x = 2y$ and $x = y^2 - 3$.

$$A = \int_{-1}^3 (2y - (y^2 - 3)) dy = 32/3$$

3. Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and $y = x + 2$ around line indicated.

a. The line $x = 3$.

$$V = \int_{-1}^2 2\pi(3-x)(x+2-x^2) dx = 2\pi \int_{-1}^2 (6+x-4x^2+x^3) dx = \frac{45}{2}\pi$$

b. The line $y = -1$.

$$V = \int_{-1}^2 [\pi(x+2+1)^2 - \pi(x^2+1)] dx = \int_{-1}^2 \pi(8+6x-x^2-x^4) dx = \frac{117}{5}\pi$$

4. Find the area of the region bounded by $y = x$ and $y = x^2 - 2$.

$$A = \int_{-1}^2 (x - (x^2 - 2)) dx = 9/2$$

5. Find the volume of the solid generated when the region bounded by $y = \ln x$, $y = 0$, and $x = e$, is revolved about the x -axis. Use the disk method. Set up the integrals but do not evaluate.

$$V = \int_1^e \pi(\ln x)^2 dx$$

6. A hemispherical tank with radius 5 m is full of water. Find the work required to pump the water out of the outlet at the top of the tank. The density of water is $\rho = 1000 \text{ kg/m}^3$.

$$W = \int_{-5}^0 \rho g \pi (25 - y^2)(0 - y) dy = \frac{625\rho g \pi}{4}$$

7. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = e^x$, $x = 0$, $y = \pi$ about the x -axis. Set up the integrals but do not evaluate

$$V = \int_1^{\pi} 2\pi y (\ln y) dy$$

8. Find the length of the curve $y = \sqrt{1-x^2}$ for $-\frac{1}{2} \leq x \leq \frac{1}{2}$.

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}, \quad ds = \sqrt{1 + \left(\frac{-x}{\sqrt{1-x^2}}\right)^2} dx = \frac{1}{\sqrt{1-x^2}} dx \quad L = \int_{-1/2}^{1/2} \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) \Big|_{-1/2}^{1/2} = \frac{\pi}{3}$$

9. Find the fluid force against the end of a water tank with cross section shown below:

$$F = \int_0^2 \rho g 2 \left(\frac{y}{2} + 1\right) (2-y) dy = \frac{16\rho g}{3}$$

10. Evaluate

a. $\frac{d}{dx} (\ln(\cos^2 x)) = \frac{1}{\cos^2 x} \cdot 2 \cos x \cdot (-\sin x) = \frac{-2 \sin x}{\cos x} = -2 \tan x$

b. $\frac{d}{dx} \left(\left(\frac{1}{x}\right)^x \right) = \frac{d}{dx} (e^{x \ln x^{-1}}) = \frac{d}{dx} (e^{-x \ln x}) = e^{-x \ln x} (-\ln x + 1) = \left(\frac{1}{x}\right)^x (1 - \ln x)$

c. Let $u = 1 + \cos x$, $du = -\sin x dx$, $\int_0^{\pi/2} \frac{\sin x}{1 + \cos x} dx = \int_2^1 \frac{1}{u} du = \ln|u| \Big|_2^1 = -\ln 2$

d. Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$, $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int 2e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$

e. $\int_0^5 5^{5x} dx = \int_0^5 e^{5x \ln 5} dx = \frac{1}{5 \ln 5} e^{5x \ln 5} \Big|_0^5 = \frac{1}{5 \ln 5} (e^{25 \ln 5} - 1) = \frac{1}{5 \ln 5} (5^{25} - 1)$

f. Let $u = \ln x$, $du = \frac{1}{x} dx$,

$$\int_1^{2e} \frac{3^{\ln x}}{x} dx = \int_0^{\ln 2e} 3^u du = \int_0^{\ln 2e} e^{u \ln 3} du = \frac{1}{\ln 3} 3^u \Big|_0^{\ln 2e} = \frac{1}{\ln 3} (3^{\ln 2e} - 1)$$