

Determine the derivative for each of the following: (Do not simplify!)

$$1. \quad y = \sqrt{3x} \qquad \frac{dy}{dx} = \frac{1}{2}(3x)^{-1/2} \cdot 3$$

$$2. \quad y = \ln(4x) \qquad \frac{dy}{dx} = \frac{1}{4x} \cdot 4$$

$$3. \quad y = 3^x \qquad \frac{dy}{dx} = 3^x \ln 3$$

$$4. \quad y = \cos(2x) \qquad \frac{dy}{dx} = -2\sin(2x)$$

$$5. \quad y = \log_5 6x \qquad \frac{dy}{dx} = \frac{1}{6x \ln 5} \cdot 6$$

$$6. \quad y = e^{-x} \qquad \frac{dy}{dx} = -e^{-x}$$

$$7. \quad y = \sin(1-2x) \qquad \frac{dy}{dx} = -2\cos(1-2x)$$

$$8. \quad y = \pi \qquad \frac{dy}{dx} = 0$$

$$9. \quad y = \sin x^3 \qquad \frac{dy}{dx} = 3x^2 \cos(x^3)$$

$$10. \quad y = \tan(2\sqrt{x}) \qquad \frac{dy}{dx} = \sec^2(2\sqrt{x}) \cdot 2 \cdot \frac{1}{2} x^{-1/2}$$

$$11. \quad y = 1/(2-3x) \qquad \frac{dy}{dx} = -(2-3x)^{-2}(-3)$$

$$12. \quad y = \sqrt[3]{5x-1} \qquad \frac{dy}{dx} = \frac{1}{3}(5x-1)^{-2/3} \cdot 5$$

$$13. \quad y = \ln x^3 \qquad \frac{dy}{dx} = \frac{1}{x^3} \cdot 3x^2$$

$$14. \quad y = 2^{x^2} \qquad \frac{dy}{dx} = 2^{x^2} \ln 2 \cdot 2x$$

$$15. \quad y = \arctan(6x) \qquad \frac{dy}{dx} = \frac{1}{1+(6x)^2} \cdot 6$$

$$16. \quad y = \sin^2 2x \qquad \frac{dy}{dx} = 2\sin 2x \cdot \cos 2x \cdot 2$$

$$17. \quad y = \sec(\pi x) \qquad \frac{dy}{dx} = \pi \sec(\pi x) \tan(\pi x)$$

$$18. \quad y = \arcsin \sqrt[4]{x} \qquad \frac{dy}{dx} = \left(1 - (\sqrt[4]{x})^2\right)^{-1/2} \cdot \frac{1}{4} x^{-3/4}$$

$$19. \quad y = e^{x^{1/2}} \qquad \frac{dy}{dx} = e^{x^{1/2}} \cdot \frac{1}{2} x^{-1/2}$$

$$20. \quad y = (7x^2 + 2)^{3/2} \qquad \frac{dy}{dx} = \frac{3}{2}(7x^2 + 2)^{1/2} \cdot 14x$$

21. Find $\frac{dy}{dx}$ for $y = e^{-x} \ln(3x)$ $\frac{dy}{dx} = \frac{e^{-x}}{x} - e^{-x} \ln(3x)$
22. Find $\frac{dy}{dx}$ for $y = \frac{x^2 + 2}{x^3 + 4x^2 + 1}$ $\frac{dy}{dx} = \frac{2x(x^3 + 4x^2 + 1) - (x^2 + 2)(3x^2 + 8x)}{(x^3 + 4x^2 + 1)^2}$
23. Find $\frac{dy}{dx}$ for $y = \tan^3 \sqrt{x^2 + 2x}$ $\frac{dy}{dx} = 3 \tan^2 \sqrt{x^2 + 2x} \cdot \sec^2 \sqrt{x^2 + 2x} \cdot \frac{1}{2}(x^2 + 2x)^{-1/2} (2x + 2)$
24. Find $\frac{dy}{dx}$ for $y = \frac{e^{-x}}{\tan x}$ $\frac{dy}{dx} = \frac{-e^{-x} \tan x - e^{-x} \sec^2 x}{\tan^2 x} = -e^{-x} (\cot x + \csc^2 x)$
25. Find $\frac{dy}{dx}$ for $y = \sqrt{x} \tan^{-1} x$ $\frac{dy}{dx} = \frac{\tan^{-1} x}{2\sqrt{x}} + \frac{\sqrt{x}}{1+x^2}$
26. Find $\frac{dy}{dx}$ for $y = (1-x)^x$ for $x < 1$ $\frac{dy}{dx} = (1-x)^x \left[\ln(1-x) - \frac{x}{1-x} \right]$
27. Find $\frac{dy}{dx}$ for $\ln(xy^2) = x^2 + y^2$ $\frac{dy}{dx} = \frac{2x - \frac{1}{x}}{2/y - 2y}$
28. Find $\frac{d^2 y}{dx^2}$ for $x^2 + y^3 = 10$

$$2x + 3y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{3y^2}$$

$$\frac{d^2 y}{dx^2} = \frac{-2(3y^2) + 2x \left(6y \frac{dy}{dx} \right)}{(3y^2)^2} = \frac{-2(3y^2) + 2x \left(6y \cdot -\frac{2x}{3y^2} \right)}{(3y^2)^2} = \frac{-6y^3 - 8x^2}{9y^5}$$

29. Find the derivative of $f(x) = \frac{1}{x+2}$, using the definition of the derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x+2 - (x+h+2)}{(x+h+2)(x+2)} \right) = \lim_{h \rightarrow 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2} \end{aligned}$$

30. Find $f''(x)$ where $f(x) = x \sin(6x)$. $f''(x) = 12 \cos(6x) - 36x \sin(6x)$

31. At which points on the curve $y = x + 2 \sin x$, $0 \leq x \leq 2\pi$ is the tangent line horizontal?

$$y' = 1 + 2 \cos x = 0, \text{ when } x = \frac{2\pi}{3} \text{ or } x = \frac{4\pi}{3}$$

32. Find an equation of the tangent line to the curve $\sqrt[3]{x} + \sqrt[3]{y^4} = 2$ at $(1, 1)$.

$$\sqrt[3]{x} + \sqrt[3]{y^4} = 2 \Rightarrow \frac{1}{3} x^{-2/3} + \frac{4}{3} y^{1/3} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\frac{1}{3} x^{-2/3}}{\frac{4}{3} y^{1/4}}, \text{ so } m_T = \frac{dy}{dx} \Big|_{(1,1)} = -\frac{1}{4} \text{ so the tangent line is}$$

$$y - 1 = -\frac{1}{4}(x - 1)$$

33. A particle moves along the x -axis, its position at time t is $x(t) = \frac{t}{1+t}$, $t \geq 0$. Find its velocity and acceleration.

$$v(t) = x'(t) = \frac{1}{(1+t)^2}$$

$$a(t) = x''(t) = \frac{-2}{(1+t)^3}$$

34. Two ships are sailing toward a very small island. One ship, the Pinta, is east of the island and is sailing due west at 15 mi/h. The other ship, the Nina, is north of the island and is sailing due south at 20 mi/h. At a certain time the Pinta is 30 miles from the island and the Nina is 40 miles from the island. At what rate are the two ships drawing closer together at that time?

$$\frac{dz}{dt} = 25 \text{ mi/hr}$$

35. Water flows into a conical tank (vertex down) at a rate of $2 \text{ ft}^3 / \text{min}$. If the radius of the top of the tank is 4 ft and the height is 6 ft, determine how quickly the water level is rising when the water is 2 ft. deep.

$$\frac{r}{h} = \frac{4}{6} \Rightarrow r = \frac{2}{3}h, \text{ so } V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2}{3}h\right)^2 h = \frac{4}{27}\pi h^3$$

$$\text{Now } \frac{dV}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt}, \text{ so } \frac{dh}{dt} = \frac{9}{8\pi} \frac{\text{ft}}{\text{min}}$$