Final Review Solution **Math 131**

Day 1:

1. $f\left(f\left(x\right)\right)=f\left(\frac{3-x}{x}\right)=\frac{4x-3}{3-x}$, domain all real numbers except $x=0$ and $x=3$

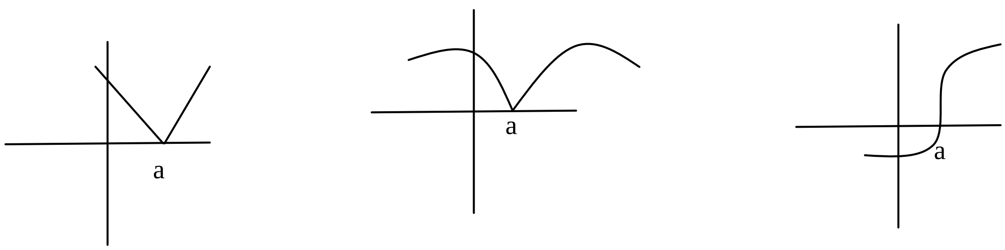
2. No, the slopes are $m_1=-2$ and $m_2=2$, respectively, so $m_1m_2=-4\neq 1$

3. False, it would only be true if f is continuous at $x=a$

4. $\frac{d}{dx}\left(f\left(x\right)g\left(x\right)\right)=f'\left(x\right)g\left(x\right)+g'\left(x\right)f\left(x\right).$

5. $\frac{dy}{dx}=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=-\frac{1}{x^2}.$

6. Sketch a function f where f is continuous at $x=a$ but f is not differentiable at $x=a$.



7. $m_{\tan}=h'\left(-3\right)=f'\left(g\left(-3\right)\right)g'\left(-3\right)=-6$ so tan line is $y-3=-6\left(x+3\right)$

8. $f'\left(x\right)=A>0$ so max occurs at $\left(b,f\left(b\right)\right)$ and min occurs at $\left(a,f\left(a\right)\right)$

9. $\lim_{x\to 0}\frac{\sin 2x}{x}\overset{L}{=}\lim_{x\to 0}\frac{2\cos 2x}{1}=.$

10. No, $g\left(x\right)$ is not differentiable at $x=2$

11. Intermediate Value Theorem

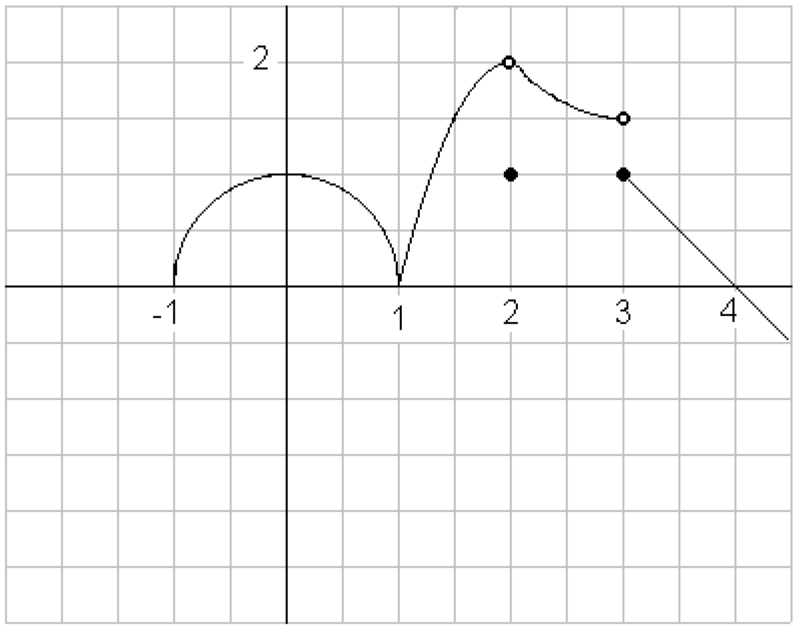
12. $y-1=\frac{4}{4-\pi}\left(x-\frac{\pi}{4}\right)$

13. Evaluate the following limits

 a. $\lim_{x\to 0}\left[\frac{1}{x^2}-\frac{\tan x}{x^3}\right]=-\frac{1}{3}$

 b. $\lim_{x\to\infty}\frac{x^2+1}{x\ln x}=DNE$ or ∞

14. Consider the function whose graph is



a. $\int_{-1}^{1} f(x)dx = \frac{\pi}{2}$

b. $f'(x) = 0$ at $x = 0$.

c. $f''(x) > 0$ for $2 < x < 3$

d. $f'(x)$ fails to exist at $x = 1, x = 2, x = 3$.

e. $f(x)$ fails to be continuous at $x = 2, x = 3$.

f. $\lim_{x \to x_o} f(x)$ fails to exist at $x = 3$

15. $R^2 = x^2 + y^2$ and $A = 2xy = 2x\sqrt{R^2 - x^2}$ for $0 < x < R$ and $A' = \frac{2R^2 - 4x^2}{\sqrt{R^2 - x^2}}$ so $A' = 0$ when $x = \frac{R}{\sqrt{2}}$ and $A'' = \frac{-2x\left(3R^2 - 2x^2\right)}{\left(R^2 - x^2\right)} < 0$ for $0 < x < R$ so maximize area when $x = y = \frac{R}{\sqrt{2}}$ and area is R^2

16. $f'(x) = \frac{27 - 4x}{3x^{2/3}}$, $f'(x) = 0$ when $x = \frac{27}{4}$ and $f'(x)$ is undefined at $x = 0$. Increasing on $(-\infty, 0) \cup (0, 27/4)$ and decreasing on $(27/4, \infty)$. Local max at $\left(\frac{27}{4}, \frac{243\sqrt[3]{2}}{8}\right)$.

Day 2:

1. Inflection point at $x=0$.

2. $\lim_{x\to 0}\frac{2x+\sin x}{x}=3$, no since $\lim_{x\to 0} y \neq \pm\infty$

3. Yes, since $f(x)=\sqrt{x}$ is continuous on $[0,2]$ and differentiable on $(0,2)$.

4. If $f'(x)=\frac{1}{2}x^2$ then $f(x)=\frac{1}{6}x^3+C$.

5. $\sum f(x)\Delta x=\sum(-2)\Delta x=-6$

6. $\int_1^2 4t^3dt=t^4\Big|_1^2=15$ units

7. The graph of $f(x)=\frac{x^2+x-1}{x-1}$ has a slant asymptote of $y=x+2$.

8. Evaluate the following:

   a. $\int \sec^2(5x)\,dx=\frac{1}{5}\tan(5x)+C$

   b. $\int_0^1 \frac{5x^2}{2x^3+1}dx=\frac{5}{6}\ln 3$

   c. $\int x(1-x)^2\,dx=\frac{1}{2}x^2-\frac{2}{3}x^3+\frac{1}{4}x^4+C$

   d. $\int_1^4 \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx=-2\left(e^{-2}-e^{-1}\right)$

9. $h(t)=\int\left(1+\frac{1}{(t+1)^2}\right)dt=t-\frac{1}{t+1}+C$, now $h(2)=5$ so $2-\frac{1}{3}+C=5$ and $C=\frac{10}{3}$. Therefore, $h(t)=t-\frac{1}{t+1}+\frac{10}{3}$ and $h(0)=\frac{7}{3}$

10. $\int_0^{\frac{3\pi}{4}} \left|\cos(2x)\right| dx = \int_0^{\pi/4} \cos(2x)\,dx + \int_{\pi/4}^{3\pi/4} -\cos(2x)\,dx = \frac{3}{2}$.

11. Find the area bounded by $y = 9 - x^2$ and $y = 0$. $\int_{-3}^{3} \left(9 - x^2 - 0\right) dx = 36$

12. Find $\frac{dy}{dx}$ for the following:

a. $\frac{dy}{dx} = 6\sec^2(6x)$

b. $\frac{dy}{dx} = 2\sin x \cos x$

c. $\frac{dy}{dx} = \frac{1}{2\sqrt{3x^5 - 4x^2}}\left(15x^4 - 8x\right)$

d. $\frac{dy}{dx} = e^{x^2 - x}(2x - 1)$

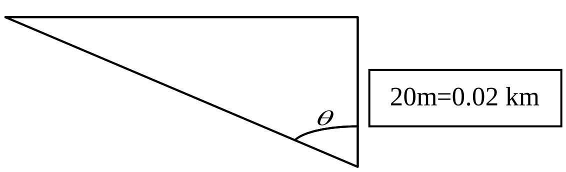
e. Use logarithmic differentiation: $\frac{dy}{dx} = x^{\ln x}\left[\frac{2\ln x}{x}\right]$

f. $\frac{dy}{dx} = \frac{-2e^x}{\left(e^x - 1\right)^2}$

g. $\frac{dy}{dx} = \frac{6}{1 + 36x^2}$

h. $\frac{dy}{dx} = \frac{2\arcsin x}{\sqrt{1 - x^2}}$

13.

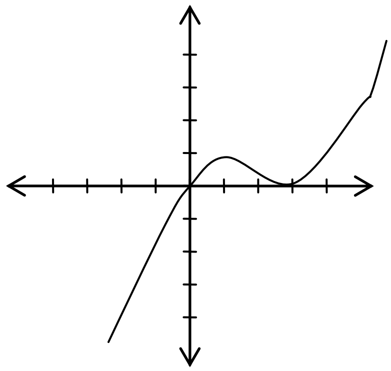


$\tan\theta = \frac{x}{0.02} \Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{1}{0.02}\frac{dx}{dt} \Rightarrow \frac{d\theta}{dt} = 10{,}000$ rad/hr

14. $f'(x) = \frac{1 - \ln x}{x^2} \Rightarrow f''(x) = \frac{-3 + 2\ln x}{x^3}$

So, f is concave up on $\left(e^{3/2}, \infty\right)$ and concave down on $\left(0, e^{3/2}\right)$ also there is a point of inflection at $\left(e^{3/2}, \frac{3}{2e^{3/2}}\right)$

15.



Evening:

1. $f(x)=\sqrt{\dfrac{x}{9-x^2}}=\sqrt{\dfrac{x}{(3-x)(3+x)}}$, domain $(-\infty,-3)\cup[0,3)$

2. $\cos\left(\arcsin\left(x^2\right)\right)=\sqrt{1-x^4}$.

3.
   a. f is continuous on $[0,3]$, since it is a polynomial.
   b. F is differentiable on $(0,3)$, since it is a polynomial.
   c. $f(0)=0=f(3)$

4. $L(x)=2+\dfrac{1}{4}(x-4)\Rightarrow\sqrt{x}\approx 2+\dfrac{1}{4}(x-4)$ for x near 4.

5. If $f'(x)=2x^2+x$ then $f(x)=\dfrac{2}{3}x^3+\dfrac{1}{2}x^2+C$.

6. No, there is a corner at $x=1$

7. If $y=\pi^5$ then $\dfrac{dy}{dx}=0$.

8. If $f(2)=1,\ f'(2)=7,$ and $h(x)=\left[f(x)\right]^3$ then $h'(2)=3\left[f(2)\right]^2 f'(2)=21.$

9. If $f(x)=\dfrac{4x-1}{2x+3}$ then find $f^{-1}(x)=\dfrac{3x+1}{4-2x}$.

10. $g\circ f=g\left(f(x)\right)=2x+4,$ for $x\geq-\dfrac{3}{2}$

11. Suppose that a particle travels along a straight line with $v(t)=t^2-2$
   a. Find the displacement of the particle for $0\leq t\leq 4$.

$$\int_0^4\left(t^2-2\right)dt=\frac{40}{3}$$

   b. The total distance traveled by the particle for $0\leq t\leq 4$.

$$\int_0^{\sqrt{2}}-\left(t^2-2\right)dt+\int_{\sqrt{2}}^4\left(t^2-2\right)dt=\frac{3\sqrt{2}+52}{3}$$

12. $\lim_{x\to\pm\infty}\frac{3x^3-2x+1}{-x^3+2x^2}=-3$, so there is a horizontal asymptote at $y=-3$ and $\lim_{x\to 0}\frac{3x^3-2x+1}{-x^3+2x^2}=\infty$ and $\lim_{x\to 2}\frac{3x^3-2x+1}{-x^3+2x^2}=\infty$ so there are vertical asymptotes at $x=0$ and $x=2$.

13. Find $\frac{dy}{dx}$ for the following:

a. $\frac{dy}{dx}=\cos(2\cos 3x)\left[-2\sin 3x\cdot 3\right]$

b. $\frac{dy}{dx}=7^{x^2}(2x\ln 7)+\frac{1}{x\ln 2}$

c. $\frac{dy}{dx}=-\frac{3x^2+y}{x+3y^2}$

d. $\frac{dy}{dx}=x^{2x}\left[2\ln x+2\right]$

14. Evaluate the following:

a. $\int\frac{\sin x}{\cos^5 x}\,dx=\frac{1}{4\cos^4 x}+C=\frac{1}{4}\sec^4+C,$ use $u=\cos x$

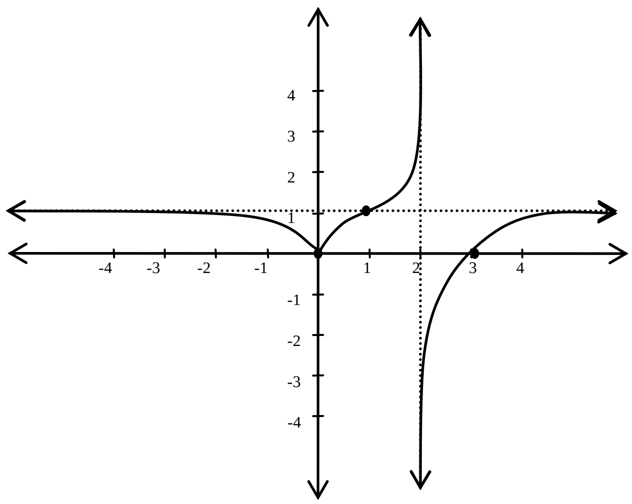
b. $\int\left(e^x-e^{-x}\right)^2 dx=\int\left(e^{2x}-2+e^{-2x}\right)dx=\frac{1}{2}e^{2x}-2x-\frac{1}{2}e^{-2x}+C$

c. $\int_0^4\frac{x}{\sqrt{x^2+9}}\,dx=\frac{1}{2}\int_9^{25}\frac{1}{\sqrt{u}}\,du=2$

15. $\lim_{x\to\infty}x\tan\left(\frac{\pi}{x}\right)=\lim_{x\to\infty}\frac{\tan\left(\frac{\pi}{x}\right)}{\frac{1}{x}}\overset{L}{=}\lim_{x\to\infty}\frac{\sec^2\left(\frac{\pi}{x}\right)\cdot\frac{-\pi}{x^2}}{\frac{-1}{x^2}}=\pi$

16. $\lim_{x\to 0}(1-2x)^{1/x}=e^{-2}$

17. Sketch a possible graph of a function with the following properties.



18. For $f(x) = x + \sin x$ for [0, 2π]

a. $f'(x) = 1 + \cos x$ f is increasing on $(0, \pi) \cup (\pi, 2\pi)$

b. $f''(x) = -\sin x$, f is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. There is an inflection point at (π, π).

19. Volume of a cylinder $V = \pi r^2 h$ and we're given $V = 1$ and $\frac{dh}{dt} = -0.1 \text{ cm/h} = -0.001 \text{ m/hr}$. Now

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} \cdot h + \pi r^2 \frac{dh}{dt} \Rightarrow 0 = 2\pi(8)\frac{dr}{dt}\left(\frac{1}{64\pi}\right) + \pi(8)^2(-0.001) \text{ thus } \frac{dr}{dt} = 0.256\pi$$

20. $V = x^2 y = 32000 \Rightarrow y = \frac{32000}{x^2}$, $S = x^2 + 4xy = x^2 + \frac{128000}{x}$, for $x > 0$, $S'(x) = \frac{2(x^3 - 64000)}{x^2}$

So $S' = 0$ when $x = 40$, $S'' = 2 + \frac{2(128000)}{x^3} > 0, \ \forall x > 0$ thus S is minimized when $x = 40$ and $y = 20$.