Final Review **Math 131**

Day 1:

1. If $f(x)=\dfrac{3-x}{x}$, find $(f\circ f)(x)$. What is the domain of $(f\circ f)(x)$?

2. Are the lines $2x+y=1$ and $2x-y=1$ perpendicular?

3. True or False: If $\lim\limits_{x\to a} f(x)=L$ then $f(a)=L$.

4. If $f(x)$ and $g(x)$ are differentiable, then $\dfrac{d}{dx}\big(f(x)g(x)\big)=$______________________.

5. If $y=\dfrac{1}{x}$, use the definition of derivative to find $\dfrac{dy}{dx}$.

6. Sketch a function f where f is continuous at $x=a$ but f is not differentiable at $x=a$.

7. If $h(x)=f\,[g(x)]$, $g(-3)=5$, $g'(-3)=2$, $f(5)=3$, and $f'(5)=-3$, find an equation of the tangent line to the graph of $h(x)$ at $x=-3$.

8. Find the maximum and minimum values of $f(x)=Ax+B$ where $A>0$ and B are constants on $[a, b]$.

9. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}=$____ .

10. Does the function $g(x)=|x-2|$ satisfy the hypotheses of the Mean Value Theorem on [1,4].

11. By the ______________________theorem, if f is continuous on the interval [a, b] and K is between $f(a)$ and $f(b)$ then $K=f(c)$ for some c in (a, b).

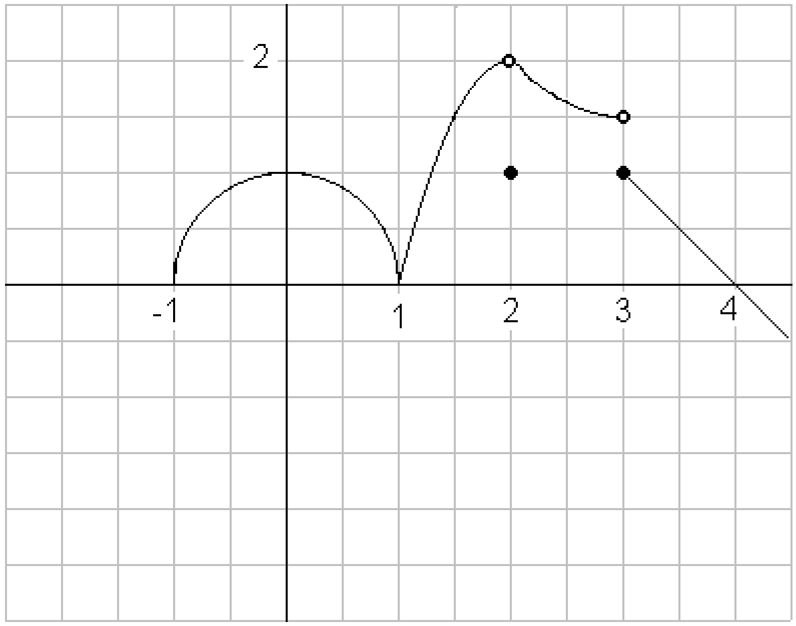
12. Find the equation of the tangent line to the graph of the equation $\tan(xy)=y^2$ at $(\pi/4, 1)$.

13. Evaluate the following limits

    a. $\lim\limits_{x\to 0}\left[\dfrac{1}{x^2}-\dfrac{\tan x}{x^3}\right]$

    b. $\lim\limits_{x\to\infty}\dfrac{x^2+1}{x\ln x}$

14. Consider the function whose graph is



a. What is the value of $\int_{-1}^{1} f(x)dx$

b. $f'(x) = 0$ at $x =$ ________ .

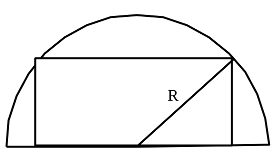
c. $f''(x) > 0$ for ____ $< x <$ ____

d. $f'(x)$ fails to exist at $x =$ ________ .

e. $f(x)$ fails to be continuous at $x =$ ________ .

f. $\lim_{x \to x_o} f(x)$ fails to exist at x = ______.

15. Find the rectangle of largest area that can be inscribed in a semicircle of radius R, assuming one side of the rectangle lies on the diameter of the semicircle as shown.



16. Let $f(x) = 27x^{1/3} - x^{4/3}$. Calculate f '(x) and use it to find all critical points of f (x). Classify all critical points and determine intervals where f (x) is increasing and decreasing.

Day 2:

1. For the graph of $y = \sqrt[3]{x}$, find the inflection point.

2. Does the graph of $y = \dfrac{2x + \sin x}{x}$ have a vertical asymptote at $x = 0$?

3. Verify that the function $f(x) = \sqrt{x}$ satisfies the hypotheses of the Mean Value Theorem on [0, 2].

4. If $f\,'(x) = \dfrac{1}{2}x^2$ then $f(x) =$ __________ .

5. If $f\left(x\right) = -2$ on [-3, 0], then the Riemann Sum for f (x) on the given interval is ______.

6. The total distance traveled during the time interval $1 \le t \le 2$ by an object with velocity $v(t) = 4t^3$ is _____ units.

7. The graph of $f\left(x\right) = \dfrac{x^2 + x - 1}{x - 1}$ has a slant asymptote of _________________.

8. Evaluate the following:

   a. $\int \sec^2\left(5x\right) dx$

   b. $\displaystyle\int_0^1 \frac{5x^2}{2x^3 + 1} dx$

   c. $\int x\left(1 - x\right)^2 dx$

   d. $\displaystyle\int_1^4 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$

9. A tree has been transplanted and after t years of growing at a rate of $\dfrac{dh}{dt} = 1 + \dfrac{1}{\left(t+1\right)^2}$ ft/yr. At two years it has reached a height of 5 ft. How tall was the tree when it was planted?

10. Evaluate $\displaystyle\int_0^{\frac{3\pi}{4}} \left|\cos\left(2x\right)\right| dx$.

11. Find the area bounded by $y = 9 - x^2$ and $y = 0$.

12. Find $\frac{dy}{dx}$ for the following:

a. $y = \tan(6x)$

b. $y = \sin^2 x$

c. $y = \sqrt{3x^5 - 4x^2}$

d. $y = e^{x^2 - x}$

e. $y = x^{\ln x}$

f. $y = \frac{e^x + 1}{e^x - 1}$

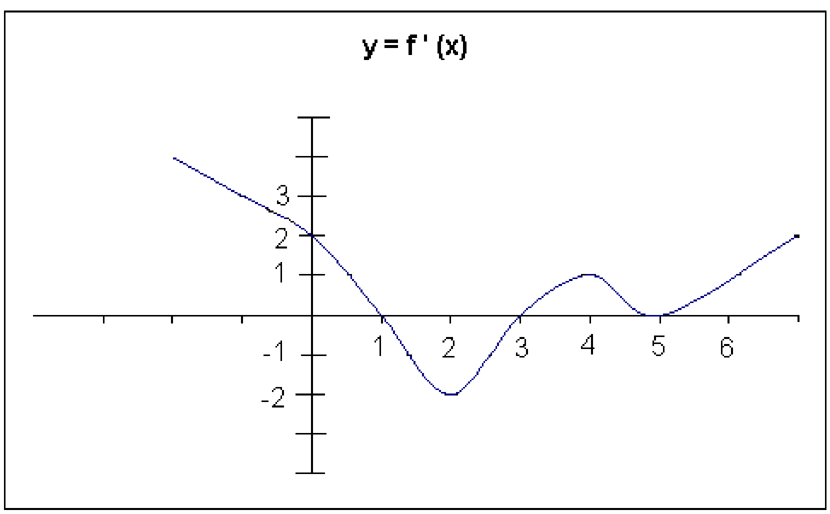
g. $y = \arctan(6x)$

h. $y = (\arcsin x)^2$

13. A race official is watching a race car approach the finish line at a rate of 200 km/h. Suppose the official is sitting at the finish line, 20 m from the point where the car will cross, and let θ be the angle between the finish line and the official's line of sight to the car. At what rate is θ changing when the car crosses the finish line?

14. Let $f(x) = \frac{\ln x}{x}$. Calculate $f''(x)$ and use it to find intervals where the graph of $f(x)$ is concave up and concave down. Find all inflection points.

15. Sketch a possible graph of a continuous function $y = f(x)$ using the graph of $f'(x)$ shown below, if $f(0) = f(3) = 0$.



Evening:

1. Find the domain of $f(x)=\sqrt{\dfrac{x}{9-x^2}}$

2. Find $\cos\left(\arcsin\left(x^2\right)\right)$.

3. Verify that $f(x)=x^3-3x^2$ satisfies the hypotheses of Rolle's Theorem on [0, 3].

4. Find the linear approximation to $y=\sqrt{x}$ at $a=4$.

5. If $f'(x)=2x^2+x$ then $f(x)=$ __________________ .

6. Is $f(x)=|x-1|$ differentiable for all x in [-5, 5]?

7. If $y=\pi^5$ then $\dfrac{dy}{dx}=$ _______ .

8. If $f(2)=1,\ f'(2)=7,$ and $h(x)=\left[f(x)\right]^3$ then $h'(2)=$ ____________ .

9. If $f(x)=\dfrac{4x-1}{2x+3}$ then find f^{-1}.

10. If $f(x)=\sqrt{2x+3}$ and $g(x)=x^2+1$ then find $g\circ f$. What is the domain of $g\circ f$?

11. Suppose that a particle travels along a straight line with $v(t)=t^2-2$
    a. Find the displacement of the particle for $0\le t\le 4$.
    b. The total distance traveled by the particle for $0\le t\le 4$.

12. Determine the horizontal asymptotes and vertical asymptotes of $y=\dfrac{3x^3-2x+1}{2x^2-x^3}$.

13. Find $\dfrac{dy}{dx}$ for the following:
    a. $y=\sin(2\cos 3x)$
    b. $y=7^{x^2}+\log_2 x$
    c. $x^3+xy+y^3=3$
    d. $y=x^{2x}$

14. Evaluate the following:

    a. $\int \frac{\sin x}{\cos^5 x}\, dx$

    b. $\int \left(e^x - e^{-x}\right)^2 dx$

    c. $\int_0^4 \frac{x}{\sqrt{x^2+9}}\, dx$

15. Find $\lim_{x\to\infty} x \tan\left(\frac{\pi}{x}\right)$

16. Find $\lim_{x\to 0} (1-2x)^{1/x}$

17. Sketch a possible graph of a function with the following properties.

Domain and Range	First Derivative	Second Derivative
$f(0)=0$	$f'(x)>0$ $0<x<1,\ 1<x<2,\ x>2$	$f''(x)>0$ $1<x<2$
$f(1)=1$	$f'(x)<0$ $x<0$	$f''(x)<0$ $x<0,\ 0<x<1,\ x>2$
$f(3)=0$	$\lim_{x\to 0^+} f'(x)=+\infty$	
$\lim_{x\to\pm\infty} f(x)=1$	$\lim_{x\to 0^-} f'(x)=-\infty$	
$\lim_{x\to 2^-} f(x)=+\infty$		
$\lim_{x\to 2^+} f(x)=-\infty$		

18. For $f(x)=x+\sin x$ for [0, 2π]

    a. Find $f'(x)$. Use it to find critical values of $f(x)$ and intervals where $f(x)$ is increasing and decreasing.

    b. Find $f''(x)$. Use it to find inflection points of $f(x)$ and intervals where $f(x)$ is concave up and concave down.

19. A circular oil slick of uniform thickness is caused by a spill of 1 m^3 oil. The thickness of the oil is decreasing at a rate of 0.1 cm/h. At what rate is the radius of the slick increasing when the radius is 8 m?

20. A box with a square base and no top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used. Verify with the second derivative test.