1. For the parametric curve $x = t^2 + 4$, $y = 6 - t$ for $-\infty < t < \infty$,
   a. $x = (6-y)^2 + 4$.
   b. Parabola, opens in the positive x direction with vertex at $(4,6)$.

2. Find an equation of the line tangent to the cycloid $x = t - \sin t$, $y = 1 - \cos t$ at $t = \pi/6$.

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\sin t}{1-\cos t}$, so the slope to the tangent line is $m_{\tan} = \left.\dfrac{dy}{dx}\right|_{t=\pi/t} = \dfrac{\sin(\pi/6)}{1-\cos(\pi/6)} = \dfrac{1}{2-\sqrt{3}}$

Also $x(\pi/6) = \dfrac{\pi}{6} - \sin\dfrac{\pi}{6} = \dfrac{\pi}{6} - \dfrac{1}{2}$ and $y(\pi/6) = 1 - \cos\dfrac{\pi}{6} = 1 - \dfrac{\sqrt{3}}{2}$ so the tan line is

$$y - \left(1 - \frac{\sqrt{3}}{2}\right) = \frac{1}{2-\sqrt{3}}\left(x - \left(\frac{\pi}{6} - \frac{1}{2}\right)\right)$$

3. Find the slope of the line tangent to the polar curve $y = 4\sin 2\theta$ at the tip of the leaves.

   The tips of the leaves are at $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ and

   $\dfrac{dy}{dx} = \dfrac{dr/d\theta \sin\theta + r\cos\theta}{dr/d\theta\cos\theta - r\sin\theta} = \left.\dfrac{8\cos 2\theta\sin\theta + (4\sin 2\theta)\cos\theta}{8\cos 2\theta\cos\theta - (4\sin 2\theta)\sin\theta}\right|_{\theta=\frac{\pi}{4}} = -1$ at

   $\theta = \dfrac{3\pi}{4}$, and $\theta = \dfrac{7\pi}{4}$ $\quad \dfrac{dy}{dx} = 1$

4. $(2, 5\pi/6) = \left(2\cos\dfrac{5\pi}{6}, 2\sin\dfrac{5\pi}{6}\right) = \left(-\sqrt{3}, 1\right)$

5. For the point with Cartesian coordinates $\left(\sqrt{3}, -3\right)$
   a. $\left(2\sqrt{3}, \dfrac{5\pi}{3}\right)$.
   b. $\left(-2\sqrt{3}, \dfrac{2\pi}{3}\right)$

6. Replace the Cartesian equation by equivalent polar equations
   a. $x + y = 4 \Rightarrow r\cos\theta + r\sin\theta = 4 \Rightarrow r = \dfrac{4}{\cos\theta + \sin\theta}$
   b. $(x-5)^2 + y^2 = 25 \Rightarrow x^2 + y^2 = 10x \Rightarrow r^2 = 10r\cos\theta \Rightarrow r = 10\cos\theta$

7. Replace the polar equation by the equivalent Cartesian equation. Then describe or identify the graph.
   a. $r = 4\csc\theta \Rightarrow r\sin\theta = 4 \Rightarrow y = 4$

b. $r = 8\cos\theta - 15\sin\theta \Rightarrow r^2 = 8r\cos\theta - 15r\sin\theta \Rightarrow x^2 + y^2 = 8x - 15y$

$$\Rightarrow (x-4)^2 + \left(y + \frac{15}{2}\right)^2 = \frac{289}{4}$$

8. Write the equation of the tangent line to the curve $r = 1 + \sin\theta$ at $\theta = 3\pi/4$

$$\frac{dy}{dx} = \frac{dr/d\theta \sin\theta + r\cos\theta}{dr/d\theta \cos\theta - r\sin\theta} = \left.\frac{\cos\theta\sin\theta + (1+\sin\theta)\cos\theta}{\cos\theta\cos\theta - (1+\sin\theta)\sin\theta}\right|_{\theta=\frac{3\pi}{4}} = 1 + \sqrt{2}$$

Tan Line: $y - \frac{\sqrt{2}+1}{2} = \left(1+\sqrt{2}\right)\left(x + \frac{\sqrt{2}+1}{2}\right)$

9. Graph the polar equation: Check with your calculator!
   a. $r = 4\sin\theta$
   b. $r = 2 + 2\cos\theta$
   c. $r = 5\cos 3\theta$

10. Find the area inside $r = 3 + 2\sin\theta$ and outside $r = 2$.

$$A = 2\left[\int_{-\pi/6}^{\pi/2} \frac{1}{2}(3+2\sin\theta)^2\, d\theta - \int_{-\pi/6}^{\pi/2} \frac{1}{2}(2)^2\, d\theta\right] = \frac{14\pi}{3} + \frac{11\sqrt{3}}{2}$$

11. Find the length of the curve $r = \sin^3\left(\frac{\theta}{3}\right),\ 0 \le \theta \le \pi$.

$$ds = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta = \sin^2\left(\frac{\theta}{3}\right) d\theta$$

$$L = \int_0^{\pi} \sin^2\left(\frac{\theta}{3}\right) d\theta = \frac{\pi}{2} - \frac{3\sqrt{3}}{8}$$

12. Find the area that lies inside both curves $r = \sin 2\theta,\ r = \sin\theta$.

$$A = 2\left[\int_0^{\pi/3} \frac{1}{2}(\sin\theta)^2\, d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\sin 2\theta)^2\, d\theta\right] = \frac{\pi}{4} - \frac{3\sqrt{3}}{16}$$

13. Find the area of the region enclosed by the inner loop of $r = \frac{1}{2} - \cos\theta$. Set up the integral but do not evaluate.

$$A = 2\int_0^{\pi/3} \frac{1}{2}\left(\frac{1}{2} - \cos\theta\right)^2 d\theta = \frac{\pi}{4} + \frac{5\sqrt{3}}{8}$$

14. Evaluate the expression and write your answer in the from $x + yi$

a. $\frac{1+4i}{3+2i} = \frac{11}{13} + \frac{10}{13}i$

b. $\left|2\sqrt{3} + 2i\right| = \sqrt{12+4} = 4$

15. Write $6e^{i4\pi/3} = 6\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = -3 - 3\sqrt{3}i$ in the form $x + yi$

16. Find the indicated power of the following using De Moivre's Theorem. Write your answer in the form $x + yi$

$$(-2-2i)^4 = \left(2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)\right)^4 = 64(\cos 5\pi + i\sin 5\pi) = -64$$

17. Find the fifth roots of 32. Sketch the roots in the complex plane.

$32 = 32(\cos 0 + i\sin 0)$

$w_1 = 2(\cos 0 + i\sin 0)$ $w_2 = 2\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$ $w_3 = 2\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$

$w_4 = 2\left(\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}\right)$ $w_5 = 2\left(\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}\right)$