Review Postulates of Quantum Mechanics

Particle in a One Dimensional Box

Define the Potential
\[ V = \begin{cases} \infty & x < 0 \\ 0 & 0 \leq x \leq L \\ L & x > L \end{cases} \]

Write the Hamiltonian Operator
\[ \hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V \]

Solve the Wave Equation
\[ \hat{H}\psi = E\psi \]

Solution for the Regions $x < 0$ and $x > L$
\[ \psi = 0 \]

Solution for Region $0 \leq x \leq L$
\[ \psi = C \cos\left(\frac{2mE}{\hbar^2}\right)^{1/2} x + D \sin\left(\frac{2mE}{\hbar^2}\right)^{1/2} x \]

Boundary Condition; at $x=0$, $\psi=0$
\[ C = 0 \]
\[ \psi = D \sin\left(\frac{2mE}{\hbar^2}\right)^{1/2} x \]

Boundary Condition; at $x=L$, $\psi=0$
\[ \sin\left(\frac{2mE}{\hbar^2}\right)^{1/2} L = 0 \]

This Means
\[ \left(\frac{2mE}{\hbar^2}\right)^{1/2} L = n \pi \quad \text{where } n = 0,1,2,3,\ldots \]
Case Where $n=0$
Resolve the Wave Equation
\[ \psi = c x + d \]
Boundary Condition; at $x=0$, $\psi=0$
\[ d=0 \]
Boundary Condition; at $x=L$, $\psi=0$
\[ c=0 \]
Implies $\psi=0$ Everywhere
\[ n=0 \text{ Not Allowed} \]

Case Where $n = 1,2,3,\ldots$
\[ E_n = \frac{\hbar^2}{8mL^2} n^2 \quad \text{where } n = 1,2,3,\ldots \]

Only Quantized Energies are Allowed
Result of Being in a "Bound" State

Back to Solution for $\psi$
\[ \psi_n = D \sin\left(\frac{n\pi x}{L}\right) \quad \text{where } n = 1, 2, 3, \ldots \]

Apply Normalization Condition

\[ 1 = \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) \, dx \]

\[ D = \left(\frac{2}{L}\right)^{1/2} \]

Final Solution

\[ \psi_n = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right) \quad \text{where } n = 1, 2, 3, \ldots \]